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ALMAGESTUM NOVUM

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English translation — Book IX, Section V: On the Harmonic System of the World (printed pp. 501–533)

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*(printed p. 501 — the opening of **Section V, "On the Harmonic System of the World."** Chapter I establishes the necessity of the section — no astronomer denies a certain harmony in the planets' intervals and motions — and supplies a long catalogue of authors on harmonic proportions, from antiquity through Kepler and Kircher, plus authors on the praises of music. Chapter II, "On the Concord of the Heavens in General," then opens with the most ancient testimony, God's words in Job 38 on the concord of heaven, expounded through various versions and the Hebrew* Nebel.*)*

SECTION FIVE — On the Harmonic System of the World

(Sectio V. De Systemate Mundi Harmonico)

CHAPTER I

On the Necessity of this Section, and on the Authors who have treated of Harmonic Proportions

[I.] No one a little more learned in Astronomy, who has contemplated the order of the heavens, would fail to acknowledge a certain Harmony in the intervals and motions of the Planets; and of what kind and how great it is cannot be neglected by one who professes an accurate treatment of the System of the World and of the heavens. But since there are many things necessary or useful for the Theory and Practice of Music which nevertheless are by no means required for this dissertation on the Harmonic System, yet, lest I should too little satisfy anyone's desire and curiosity, I shall at least point out not a few Authors, from whom, as from springs, he may perhaps slake this thirst, importunate though it be.

[Margin: The chief Authors on Harmonic Proportions.]

They were: **Aristotle** (section 19 of the *Problems* — and there **Blancanus**, on the Mathematical passages of Aristotle, from marginal number 359); **Nicomachus** (in the *Enchiridion of Harmonics*); the *Musical fragments of Heraclides Ponticus* in Athenaeus; **Aristoxenus** (the 3 books of the *Elements of Harmonics*, Antonio Gogava translating); **Euclid** (in the *Elements of Music*); **Panætius** (the book *On Geometric and Musical Ratios*); **Vitruvius** (bk. 5, ch. 4 — and there **Daniele Barbaro** and **Wilhelm Philander**); **Pliny** (bk. 2, ch. 22, and his interpreters there); **Plutarch** (the opuscule *On Music*, with the exposition of Carlo Valgulio); **Ptolemy** (the *Harmonics*, with the 3 books of notes, Antonio Gogava translating); **Porphyry** (on Ptolemy's *Harmonics*); **Cassiodorus** (*On the disciplines*, under the title "On Music"); **St. Augustine** (*On Music*); **Boethius** (the book *On Music*, with the exposition of Franchino Gaffurio and of Lefèvre d'Étaples); **Apuleius** and **Psellus** (*On Music*); **Censorinus** (*On the Birthday*, ch. 11); **Isidore** (*Origins*, bk. 3, in the chapter on Music); **Martianus Capella** (bk. 9 of the *Philology*); **Bede** (the opuscule *On Music*); **Macrobius** (bk. 2 *On the Dream of Scipio*, chs. 1–5); **Guido of Arezzo** (in the *Introduction to Music*); **Berno**, Abbot of Cluny; John Fro[…]; **Otho the Theoger**, Bishop; Othobonus[…] in the books *On Music*; **Josquin des Prez** (otherwise called Iusquinus); likewise **Jean Richafort**, **Nicolaus** [Wollick/Giselin], **Adam of Fulda**, …

Jacob Obrecht the Fleming; **Sixt Dietrich**; **Gerhard à Salice**; **Andreas Sylvanus**; **Nicolaus Craën**; **Vincenzo Lusitano** (in the *Musical Compendium*); **Orazio Tigrini**; **Pietro Pontio**; **Stefano Vanneo**; … **Giovanni and Giorgio Valla**; **Alypius** (in the *Isagoge of Music*); **Artusi** (*On the imperfection of modern Music*); **Damião de Góis**; **Gregor Meyer**; **Giovanni Spataro** of Bologna; **Nicola Vicentino**; **Theodoricus** of Augsburg; **Girolamo Diruta**; **Heinrich Glarean** (in his *Dodecachordon*); **Lodovico Fogliano** of Modena (in the *Musica Theorica*); **Franchino Gaffurio**; **Gioseffo Zarlino** (in the *Harmonic Institutions*); **Vincenzo Galilei** (in the *Dialogues on Music*); **Marsilio Ficino** (in the *Compendium on Plato's Timaeus*, chs. 27–35); **Scipione Cerreto** of Naples; **Giovanni Pierluigi da Palestrina**; **Lodovico Zacconi**; **Aloysius Denticus**; **Felice Anerio**; **Francisco Salinas** (Doctor of the School of Salamanca); **Sebastián Raval**; **Illuminato Aiguino**; **Agostino Pisa** (*On the percussion* of Music); **Clavius** (in bk. 5 of the *Elements*, where on Harmonic proportions); **Robert Fludd** (*On the Microcosm and Macrocosm*, each of which he contends to have been framed according to Harmonic laws); **Johannes Kepler** (in the five books of the *Harmonics*, and in the *Mysterium Cosmographicum* ch. 12, and in the *Epitome of Copernican Astronomy* bk. 6, from pp. 840 and 900); **Pierre Hérigone** (vol. 5 of the *Mathematical Course*, from p. 573, and more fully in the same vol. 5, p. 802, on Euclid's *Music*); **Marin Mersenne** (on ch. 4 of Genesis, from verse 21, and in the paralipomena of the same work from p. 1854); **Mario Bettini** (*Apiarium* 10, Proludium 1); and most recently and most learnedly, **Athanasius Kircher** (in the 10 books of his *Musurgia*).

[Margin: The chief Authors on the Praises of Music.]

[II.] But concerning the Praises of Music, and its admirable power for many things — but especially for stirring up and composing the motions of the soul, etc. — [see]: **Plutarch** (the opuscule *On Music*); **Pietro Vanneo** (*On the power of Music*); **Martianus Capella** (bk. 9, *On the Nuptials of Philology and Mercury*, ch. 1); **Zarlino** (part 1 of the *Harmonic Institutions*, chs. 2–7, and part 2, chs. 4–9); **Mersenne** (on ch. 4 of Genesis, verse 21); and **Athanasius Kircher** (passim, but especially bks. 9 and 10 of the *Musurgia*).

CHAPTER II

On the Concord of the Heavens in General

[Margin: The concord of the heaven from the Sacred letters.]

[I.] We have a most ancient testimony concerning the concord of the Heavens from the very Founder and Harmost [Tuner] of the Heavens, GOD Himself, when He thus speaks with Job [Job ch. 38]: *"Who shall declare the reason of the Heavens, and who shall make the concord of heaven to sleep?"* — which passage, although various men interpret variously, yet very many refer to the harmony that shines forth in the motions, and in the intervals and arrangement, of the stars. And so the Chaldee paraphrast: *"Who established the stars of heaven in wisdom?"* — or, who established the weeks of heaven in wisdom, and who dwelt in the revolution of heaven? But the Tigurine [Zurich] version: *"Who shall dictate to the Aether by his wisdom what it should do, and who shall tilt the bottles of heaven?"* But Pagninus: *"Who has numbered the heavens in wisdom, and the bottles of heaven who [shall stay]; the waters which descend from heaven, who made to rest?"* Again, Cajetan and Vatablus interpret thus: *"Who so prudently made the heavens in the number in which they are?"* And Philippus [of Aquino], in our Pineda upon that passage, interprets the concord of heaven as the arrangement and beauty of the heavens, for which men or Angels praise God; and thus "concord" is there taken objectively, for the object exciting the rational creature to the song and praises of God — in the manner it is said in Psalm 18[19]: *"The heavens declare the glory of God."*

[Margin: The praises of the Cock.]

But St. Thomas and Lyra add that this saying can be referred to the preceding verse, in which God had said: *"Who gave the cock understanding?"* — for He subjoins: *"Who shall declare the reason of the heavens?"* —

as if to say: who shall declare to the cock the proportion of the celestial motions, that thence it might discern the appointed hours for crowing? Since indeed the cock, by its own crowing, seems to mark off the four watches of the night at each set of three unequal hours, but most of all the fourth watch; on which account see the praises of the cock in Pliny (bk. 10, ch. 21). Now the word *Concentus* [concord], in Hebrew, is *Nebel*, from the root *Nabal*, which means "to flow down," and, by synecdoche, the vessel by which flowing water is caught and carried — of which kind are wineskins; but since from vessels of this kind certain…

[…continues on p. 502 (PDF 537) with the catchword "aliqua" — "…certain [musical instruments] were made," and the discussion of the Hebrew Nebel as a musical instrument continues.]

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*(printed p. 502 — **Chapter II** continues its survey of testimonies to world-harmony. The philology of the Hebrew* Nebel *and related scriptural passages (Joshua 10, Wisdom 19) is completed; then the Fathers are marshalled — Augustine, Ambrose, Philo, Boethius, Isidore, Bede, Anselm — on the heavens' harmony and Boethius's threefold division of music; finally the pagan witnesses, chiefly Pythagoras, Plato, and Cicero's Dream of Scipio with its full music-of-the-spheres passage, and Macrobius on the nine Muses.)*

…certain musical instruments [were made of such vessels], it is therefore taken for a Psaltery, or *Nablum* [Nabla], from the same root *Nabal*; and so for "concord," as the Vulgate has it; or as Symmachus has it: *"Who shall adorn the organs [instruments] of heaven?"*; or, with St. Augustine: *"Who shall tilt the organs of heaven?"* Yet most of the Rabbis, in Pineda, interpret [the "bottles of heaven"] as clouds full of water, or wineskins; and the Royal Bible [Biblia Regia], "the flowing-down of the heavens" — especially when the rain-clouds [pour down] rains with a great sound, which God suddenly makes to fall silent when He brings on fair weather.

[Margin: The symphony of the elements.]

Besides, that [verse] of Joshua, ch. 10 — *"Sun, be not moved,"* and *"the Sun stood still"* — in Hebrew is *dom* and *vajdom*, namely (as the Royal Bible has it) *"be silent"* and *"it was silent"*: where, metaphorically, by "sound" or "voice" motion is signified, and by "silence," the rest of the heaven; for the fabrications which the Rabbis there invent our [Father] Serarius has sufficiently refuted on that passage. And those words of Wisdom 19, verse 17[18], also make for our point: *"For while the elements are turned among themselves, as in an instrument the sound of the quality is changed, and all keep their own sound [time]; whence it can be certainly estimated from the very sight"* — which passage our [Fathers] John Lorin and John Pineda excellently interpret [of the harmony of the elements]; and it is most of all illustrated by what Fr. Athanasius Kircher (in bk. 10 of the *Musurgia*, on the symphony of the elements, from p. 367) adduces, among many other things — the Tetrachord of Orpheus, by whose testimony (with Bryennius) *Hypate* referred to Earth, *Parhypate* to Water, *Paranete* to Air, and *Nete* to Fire.

[Margin: The Harmony of the World and the heavens, from the Fathers. — St. Augustine.]

[II.] Nor indeed do the Fathers dissent from the Scriptures. **St. Augustine**, on the harmony of the whole world (bk. 11 of the *City of God*, ch. 18), said: *"God has graced the order of the ages as a most beautiful poem out of certain, as it were, antitheses; and just as contraries set against contraries render the beauty of speech, so, by a certain eloquence not of words but of things, the beauty of the age is composed by the opposition of contraries."* He too, on that [verse] of Psalm 148, "his praise is above heaven and earth," says: *"What is the praise of God? Is it that He Himself confesses? No, but that all things cry Him aloud — the beauty of all things is, in a manner, a confessing of God. Heaven cries, 'Thou madest me, not I myself'; earth cries, 'Thou foundedst me, not I myself,'"* etc.

[Margin: St. Ambrose. — Philo.]

St. Ambrose agrees with him (bk. 4 of the *Hexameron*, and in the preface upon the Psalms), saying: *"The powers of the heavens praise the Lord, they sing psalms to Him; the stars too sing psalms to the Lord."* The heaven's more express utterance [tells] that it is engaged in a certain perpetual sweetness of concord, so that its sound is heard to the farthest parts of the earth, where there are certain secrets of nature; nor does this seem foreign to the reason of nature. But before him, **Philo** (in the book *On Dreams*) most eloquently: *"The heaven, by its perpetual concord, renders a most sweet harmony; which, if it could reach our ears, would excite in us overpowering loves and a mad desire for them — stimulated by which, we would forget the other necessary things, and, leaving aside food and drink, as candidates for immortality, refreshed by the divine concords and songs: such as, when Moses had heard them, he is said for forty days and as many nights to have tasted neither bread nor water."* And a little after: *"The heaven, the archetypal instrument of Music, seems to have been elaborated for no other reason than that hymns might be skillfully and musically sung to the Parent of things."*

[Margin: Boethius.]

Let there succeed to Philo the most knowing of musical proportions, **Severinus Boethius** (bk. 1 *On Music*, ch. 2), where he divides Music threefold — namely into Mundane, Human, and Instrumental — the Mundane comprehending the admirable order and consonance of divine providence, especially in the disposition of the heavens; and among other things he says: *"How can it be that so swift a machine of heaven should be moved with a silent and noiseless course, even though that sound does not reach our ears?"*

[Margin: St. Isidore. — Bede.]

Isidore (bk. 3 of the *Origins* [Etymologies], ch. 16) says: *"Without Music no discipline can be perfect, for nothing is without it. For the World itself is said to be composed of a certain harmony of sounds, and the heaven itself revolves under the modulation of harmony."* But much more does **Bede** write (vol. 1, in the *Musica Theorica*, p. 406), saying: *"While the celestial Music is made up of subtler [bodies], it is rendered most sonorous, without any incongruity, [and] is heard from the higher [bodies] down to the lower; for it is secretly poured forth [into the ears], although on account of habit we do not perceive it — just as those who dwell near the κατάβαθμόν [katabathmon], that is, near the descent [cataract] of the Nile, lack, on account of the magnitude of the sound, the sense of hearing it. But if anyone were born in another world (if that were possible) and should afterward come into this one, he would hear it without any hindrance, and it would please him beyond measure."*

[Margin: St. Anselm. — The poet Licentius.]

Similar things has **St. Anselm** (bk. 1, *On the Image of the World*): *"The orbs of the seven heavens revolve with a most sweet harmony, and by their circuit the sweetest concords are produced. This sound therefore does not reach our ears, because it is made beyond the air, and its magnitude exceeds our narrow hearing."* To these is to be numbered **Licentius**, the Christian Poet, a disciple of St. Augustine, who (as in St. Augustine's Epistle 39, and in Sixtus of Siena, bk. 5 of the *Library*, annotation 105), speaking of GOD, thus sang:

He fitted numbers to the heavens; and bade them ply / Sonorous measures, and drive on like choral dances.

[Margin: The praise of the number Seven.]

[III.] Let us now hear the outsiders [the pagans], of whom the most ancient, **Pythagoras**, is said to have attained such fame for this reason — that he taught the whole World to be harmonically arranged (as Athenaeus asserts, bk. 14, ch. 13); but also **Plato** and **Archytas**, of whom **Plutarch** (at the end of the opuscule *On Music*) said: *"But now, friends — what is the highest of all, and what most declares that Music is to be cultivated, has been passed over by you. For Pythagoras, Archytas, Plato, and the rest of the ancient Philosophers proclaimed that the motions of all things and the revolutions of the stars can neither come to be nor subsist without Music: for they contend that GOD the Maker fashioned all things with harmony."*

Certainly **Plato** (bk. 10 of the *Republic*) said that upon each of the celestial orbs sits a Siren; and from the Platonic springs flowed that [saying] of **Cicero** (bk. 1 of the *Tusculans*): *"When Archimedes bound together in a sphere the motions of the Moon, the Sun, and the five wandering [planets], he accomplished the same thing as He who, in Plato's Timaeus, built the World — GOD — namely, that one revolution should govern motions most unlike in slowness and swiftness."* But more fully in the **Dream of Scipio**, where Scipio himself says:

> *"'What,' I said, 'what is this sound, so great and so sweet, that fills my ears?' And [Paulus, his father], answering: 'This is that which, produced by the impulse and motion of the orbs themselves — separated by unequal intervals, yet distinguished in due proportion — tempering high notes with low, evenly produces various concords. For such great motions cannot be sped on in silence; and nature brings it that the extremes sound, on the one side low, on the other high. For which reason that highest course of the star-bearing heaven, whose revolution is swifter, is moved with a high and rapid sound; but this lowest, the Lunar, with the lowest. For the earth, the ninth, remaining immovable, always clings in the lowest seat, embracing the middle place of the world. But those eight courses, in two of which is the same force [pitch], produce seven sounds distinguished by intervals: which number is well-nigh the knot of all things. Learned men, imitating this with strings and songs, have opened for themselves a return to this place — as have others who, with outstanding talents, have cultivated divine studies in human life. The ears of men, filled with this sound, have grown deaf; nor is there in you any duller sense: just as, where the Nile rushes down from the highest mountains to the place called Catadupa [the Cataracts], the nation that dwells near the place lacks, on account of the magnitude of the sound, the sense of hearing. But so great is this sound, [made] by the most rapid revolution of the whole world, that the ears of men cannot take it in; just as you cannot gaze upon the Sun with eyes set against it, and your sight and sense are overcome by its rays.'"*
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> *[Margin: Macrobius.]*

Subscribing to whom, **Macrobius** (bk. 2, *On the Dream of Scipio*, ch. 1) [treats] of that sound: *"For from the very circuit of the orbs a sound must necessarily be born; because the struck air itself emits a crash; by the very violent collision of the two bodies… From these things, by an unconquerable reasoning, it is gathered that sounds proceed from the revolution of the spheres, and that sound must arise from motion, and that the proportion which is in divine things becomes the cause of the modulation."* — This, **Pythagoras**, first of all the men of the Greek nation, conceived in his mind; and he goes on to narrate by what reasoning Pythagoras, having heard the strokes of hammers in the workshop of an ironsmith, discovered the harmonic proportions. And in ch. 3: *"Hence [arises], from the revolution of the celestial spheres…"*

> *[Margin: The ninth Muse.]*

Plato, in his Republic, when he treated of the revolution of the spheres, said that upon each sits a Siren — signifying that song is produced by the motion of the spheres; for "Siren," by the Greek interpretation, sounds [as] "singing to God." To this point look also the Fables [Apologues] of the nine Muses: the eight sounds of the spheres, and one greatest concord which consists of all [the rest]. The Theologians too [hold] the musical songs of the spheres, and that ninth Muse which consists of all, to be…

> *[…continues on p. 503 (PDF 538) with the catchword "esse" — the discussion of the nine Muses and the spheres' concord continues, still within Chapter II.]*

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*(printed p. 503 — **Chapter II** continues and turns to its central question. The survey of testimonies to celestial harmony concludes with the nine Muses, Pliny, Ptolemy, and others; then the chapter asks whether the heavens' "concord" is proper or metaphorical, under the heading whether the celestial bodies produce a real sound by their motion. The debate is set out: the affirmative ascribed to Pythagoras and reported by Aristotle (who judges it impossible), with lists of authorities for and against, including St. Basil and St. Ambrose.)*

…[the Theologians hold the spheres' songs, and that ninth Muse which consists of all, to] be [the song of the world]. He adds, moreover, from Hesiod, that the eighth of the Muses, who presides over the star-bearing sphere, is Urania; and the ninth is the concord from the sound of all the spheres, and is called Calliope from the beauty of [her] voice; and he concludes: *"That the Muses are the song of the world, even the Etruscans know, who called them Camenae, as it were 'the singers,' from singing [canere]."* Therefore the Theologians too, approving that the heaven sings, employed musical sounds in [their] sacrifices, etc. And soon after: *"In the very hymns of the Gods also, the meters [were composed] by strophe and antistrophe in melodious verses; so that by the strophe the direct [motion] of the star-bearing orb [was imitated], and by the antistrophe the diverse retrograde course of the wandering [planets] was signified — from which two motions the first hymn in nature, to be dedicated to God, took its beginning."* He alludes to the passage of Macrobius and of Plato — that hemistich of Angelo Poliziano: *"To each his own harmless Siren."*

[Margin: The strophe [is] an imitation of the twofold motion of the heaven.]

Furthermore, **Pliny** (bk. 2, ch. 3), speaking of the motion of the stars, says: *"Whether there be an immense sound — and therefore one easily exceeding the sense of [our] ears — from the assiduous whirling of so great a mass rotated, I would not indeed easily say: no more, by Hercules, than [whether there be] the jingling of the stars whirled around together and of their orbs rolling them; or whether it be a concord of sweet and incredible suavity. To us who act within, the world glides by in silence alike by day and by night."* In the same book, ch. 22, he reports the Pythagorean distribution of the celestial intervals according to the Musicians' canons, of which below.

[Margin: Psellus. — Censorinus. — Dorylaus. — Martianus Capella.]

Michael Psellus (at the beginning [of his work] on Music) says: *"The ancients say that Music contains all things."* But more ancient than he, **Censorinus** (*On the Birthday*, ch. 11, toward the end), speaking of Pythagoras, said: *"Besides many other things which the Musicians treat, he reported that [Pythagoras] showed the stars and this whole World to be εὐαρμόνιον [well-harmonized]."* Wherefore **Dorylaus** wrote that the World is the organ [instrument] of God; the rest concerning Pythagoras, from Censorinus, let us reserve for ch. 7, num. 2. Accordingly, not incongruously, **Martianus Capella** (*On the Nuptials of Philology and Mercury*), after the 8th book, which is wholly on Astronomy, immediately subjoined the 9th, which is on Music and is so entitled, where he introduces Harmony thus speaking of herself: *"Long ago indeed [I left] the earth-born… I strike the starry orbs of heaven… since the very rapidity of the machine [of heaven] going its round sings together, and acknowledges a melody fitting to its all-sounding pulses."*

[Margin: Ptolemy.]

But we have almost forgotten our [own author] **Ptolemy**, who (bk. 1 of the *Harmonics*, ch. 2) thus lays down: *"It is the Harmonist's aim everywhere to conserve the rational positions of the canon [scale], in no way repugnant to the senses, according to the opinion of the majority; as it is the Astronomer's [aim] to conserve the consonant positions of the celestial motions, the revolutions being observed."* And in bk. 3, ch. 8, he refers the configurations [aspects] of the celestial [bodies] to musical laws; and in chs. 9, 10, 11, and 12, he compares the celestial motions to harmonics.

[Margin: Cœlius Rhodiginus. — Pietro Gregorio of Toulouse. — Zarlino.]

Cœlius Rhodiginus (bk. 5 of the *Ancient Readings*, ch. 25) asks, or wonders: *"In what way it comes about that the rotated whirling of such great orbs is carried in a silent and noiseless circuit — even though, we hearing it less, the wondrous pleasantness of the sounds passes [us] by?"* Similarly **Pietro Gregorio** of Toulouse (bk. 12 of the *Syntaxis*, ch. 7), defining celestial Music, says: *"Celestial Music is that which arises from the ordered disposition of the orbs and Planets, by a concordant and recurring motion, in a definite space."* **Joseph Zarlino** too (part 1 of the *Harmonic Institutions*, ch. 6) brings forward, according to his erudition, many things concerning the sound of heaven — though unheard by us on account of habituation —

and concerning the harmony in the motions and distances of the heavens.

It now remains, from what has been said, to settle the question whether the sound and concord of the heavens is to be taken properly, or rather metaphorically.

Whether the Celestial bodies produce a Real Sound by their motion?

[Margin: The argument for the real sound of the heavens.]

[IV.] The affirmative opinion is attributed to Pythagoras by Macrobius, as we have already seen (bk. 2, on the Dream of Scipio, ch. 3); but much earlier by **Aristotle**, who treats this very controversy in bk. 2 *On the Heaven*, from text 52. For at text 52 he says: *"It seems to some necessary that a sound be made by the carrying-along of such great bodies; since [a sound] arises from the carrying-along of the bodies which are among us, which nevertheless have neither such a mass nor such a velocity as do the Sun and the Moon. How then, when so many stars, so great in multitude and magnitude, are borne along with such velocity, is it impossible that some immense sound be made? Nor does it stand in the way that the sound is not perceived by us; for the cause is, that straightway from [our] birth this sound is present and occupies the ears, so that — by the lack of comparison with its contrary, silence — it is not manifest; since the discernment of sound and silence is mutual."* He adds the likeness from the ears of smiths, accustomed to the sound — alluding to that workshop in which Pythagoras, by the hearing of hammers, learned the harmonic proportions (on which see Macrobius, bk. 2 on the Dream of Scipio, ch. 1). These [words] of Aristotle are at the end of text 52.

Wherefore: *"just as to smiths, on account of habituation, nothing seems [to be heard], the same happens to all men: these things, which were said from the beginning, are wittily and musically said; [but] it is impossible that this should really be so."* — This is at text 54, where Aristotle himself reports and rejects the Pythagorean opinion (he did not, however, name the earth; namely at text 52, saying that it was "wittily and excellently said by those who affirmed this," yet not true).

[V.] The same [persuasion] Censorinus most anciently attributes to Pythagoras (in the book *On the Birthday*, ch. 11), which we shall adduce below (ch. 7, num. 2). This persuasion of Pythagoras concerning celestial Music — to speak with the Conimbricenses (*On the Heaven*, ch. 9, q. 1) — occupied the minds of very many by its sweetness, so that most [philosophers] asserted, in particular, that a sound is excited from the mutual collision of the celestial bodies, which we do not perceive — whether on account of distance, or of habituation, or because our hearing has been dulled already from infancy. Among whom [who held this opinion], openly indeed, were: **Philo** (the book *On Dreams*) and **Cicero** (in the Dream of Scipio); and — though he may seem to doubt — **Macrobius** (bk. 2 on the Dream of Scipio, chs. 1 and 3); **Martianus Capella** (bk. 9 on the Nuptials of Philology); **Bede** (in the *Musica Theorica*); **Anselm** (bk. 1 *On the Image of the World*); **Cælius Rhodiginus** (bk. 5 of the *Ancient Readings*, ch. 25); and **Joseph Zarlino** (part 1 of the *Harmonic Institutions*, ch. 6), as is plain from their words reported under numbers 2 and 3; and to the same opinion are favorable **St. Ambrose** (in the preface on the Psalms), **St. Isidore** (bk. 3 of the *Origins*, ch. 16), **Plutarch** (in the opuscule on Music), and finally reason itself, drawn from those things which we experience in other bodies.

[Margin: The Authors against the real sound of the heavens.]

Yet the contrary opinion — denying a real sound from the heavens — together with the Philosopher [Aristotle], the Peripatetics have followed: Aristotle himself from text 53 to 56; the **Conimbricenses** (2 *On the Heaven*, ch. 9); **Amici** (tract 5 *On the Heaven*, q. 6, dub. 10); the **Abulensis** (on the chapter of Joshua, q. 14); **Vallés** (*On Sacred Philosophy*, ch. 36); **Salinas** (bk. 5 of the *Harmonics*, chs. 4 and 8); **Serarius** (on ch. 10 [of Joshua], q. 13); **Lorin** (on Psalm 18[19], at that verse *"their sound went out into all the earth"*); likewise **Sixtus of Siena** (bk. 5 of the *Holy Library*, annotation 105); and [the commentators] on ch. 38 of

Job, verse 37; and not a few of the Fathers — including **St. Basil** (in the scholia on Psalm 18[19]), where, expounding that [verse] *"There are no speeches nor languages,"* he subjoins: *"they emit no speech, nor utter any word, but by the showing-forth of their order they call the whole earth to the praise of God"*; and in Homily 1 of the *Hexameron* he calls the Pythagorean opinion a "sorcerous imposture," and "flabby with rot."

[Margin: St. Ambrose [against].]

But more fully **St. Ambrose** (bk. 2 of the *Hexameron*, ch. 2): when he had reported the opinion of the Philosophers, who affirmed that a sweet sound is rendered by the impulse and motion of the celestial globes but is not perceived by us — just as neither [is] the roar of the Nile [perceived] by the dwellers at the cataracts — he subjoins: *"But the truth itself easily answers these things. We who hear thunders, generated by the collision of clouds [— why do we not also hear] the revolutions of the orbs, which surely, the greater [they are], so much the more vehement sounds would they excite, just as they are reckoned [to move] by [a greater] motion?"* — Although this holy Doctor brings forward another reason for [the imperceptibility of] this imperceptible sound, nor does he dissolve [the difficulty], while he sub-…

[…continues on p. 504 (PDF 539) with the catchword "sub-" — St. Ambrose's further reasoning, and the resolution of the question, within Chapter II.]

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*(printed p. 504 — **Chapter II** concludes, then **Chapter III** begins. The case against a real celestial sound is finished: Ambrose's final-cause reason (the sound withheld lest men abandon their business), Aristotle's physical arguments from* On the Heaven*, and additions from the Conimbricenses, Riccioli, and Kepler, who denies any sounds in heaven. Chapter III, "On Proportions, and especially on Harmonic ones," then opens the mathematical groundwork with definitions of part, ratio, and proportion, and the five genera of rational proportion, beginning with the Multiplex and Superparticular.)*

…he subjoins: *"They add, besides, that this sound therefore does not reach the earth, lest men, captivated by its sweetness and by the charm which that most swift motion of the heavens produces from the Eastern parts to the West, should abandon their proper business and labors, and all things here remain idle, [the human soul being carried away] to the celestial sounds by an ecstasy of the mind."* — Which reason, drawn from the final cause, we have already reported from Philo above. The same **St. Ambrose** (in the book *On Isaac and the Soul*), at that version of Aquila, *"Sounding like the Sun,"* hints that by that revolution of the celestial axis, and by the course of the Sun and Moon and stars, the song of the globes is expressed. By which [words] he too seems [to be one] of ours; who, since [the opinion] finds no credence, yet on account of the charm of its sweetness is not rejected outright. And in the preface to the Psalms of David he says: *"The discourse of certain men holds that the very axis [of heaven] is engaged in a certain perpetual sweetness of concord, so that, if habituation permitted, its sound would be heard in the farthest parts of the earth,"* etc. — from which it is clear that he speaks [in reporting] another's opinion. Besides these, **St. Irenaeus** (in his work *Against Heresies*) and **St. Epiphanius** (bk. 1 of the *Panarion*) reckon this too among the heresies of the Marcosians [the followers of Marcus]: that they ascribed a real sound of the stars to the heavens.

[Margin: Arguments against the real sound of the heavens.]

[VI.] But the Arguments of Aristotle are these (bk. 2 *On the Heaven*, texts 53 and 54): *"If [the bodies] of the heaven emitted as great a sound as befits them, it would come about that, although we should not perceive it by the sense of hearing, yet by touch, so to speak, we should perceive it; for sounds, when they exceed [measure], destroy even inanimate bodies — as the sound of thunder shatters certain stones, or makes them tremble and shakes them. Since, then, we perceive no such sound either by hearing or by any other sense, we say, more reasonably, that there is no [such] sound."* Again, at texts 54, 55, and 56, he teaches that the stars themselves do not produce a sound, because they are not moved by themselves but by the motion of their

orbs (as a sailor by the motion of his ship); and that the orbs themselves are not in air or in fluid Fire — [that is], in a medium suited to sound; and finally, that in that which is carried [smoothly] and makes no stroke [blow], it is impossible that a sound be made. For not any mutual rubbing of two bodies suffices for sound, especially if their surfaces are most polished; but there is required a cutting of air or water or of some intercepted fluid body, struck by an interrupted [percussive] motion.

The **Conimbricenses** add that it would come about that the sound of the heavens would be perceived by those who, having for many years lacked the use of hearing (the organ being corrupted by some disease), [should] recover their hearing [— if there were such a sound]. I add, for my part, that just as the perpetuity of light does not hinder the eye, whereby it should discern it the less, so the perpetuity of sound would not hinder [the ear], so that we should not hear the sound; nor for that is silence or rest required. It seems, however, [a thing to be examined], whether any sound at last, in the fluid Aether, [reaching] as far as the sphere of the Fixed [stars], could be produced — whether from the Firmament itself, or from the bodies of the Planets themselves — [and whether it] could reach us: both on account of the thinness of the Aether itself, which yields to their motion with almost no resistance, and on account of their huge distance from us. But **Kepler**, who thinks the sphere of the Fixed [stars] immobile, says thus of the others (bk. 5 of the *Harmonics*, ch. 4): *"Now there are no sounds in the heaven, nor is the motion so turbulent that a shriek is elicited from the friction of the celestial breeze."*

CHAPTER III

On Proportions, and especially on Harmonic ones

[Margin: Aliquot and aliquant part.]

[I.] A **Part**, according to Euclid (bk. 5), is a magnitude which measures a greater magnitude as [a part of] the whole; and if it measure [it] perfectly — so that, replicated some number of times, it equals its whole — it is called an *aliquot* part, as 2 with respect to 8. But if, when replicated, it exceeds the whole or falls short — as 2 with respect to 9 — it is called *aliquant*; for the binary [2], taken four times, falls short of the nine, but taken five times exceeds it.

[Margin: What are Ratio and Proportion?]

A **Ratio** is the mutual relation, according to quantity, of two magnitudes of the same kind. **Proportion**, however — which the Greeks call ἀναλογία [analogia], and some Latins "Proportionality" — [is the likeness of ratios]. [A proportion] is such, for example, between [a line] of six palms [and one of four]; for it can be expressed by numbers, so that such a proportion is between two commensurable [magnitudes], or [magnitudes] having a common aliquot part which can exactly measure each of them — of which kind [of measure] is the unit of an integer number; and therefore a **rational** proportion cannot [exist] except between numbers, or [magnitudes] reducible to numbers, as between 3 and 6.

[Margin: Rational and irrational proportion.]

But an **irrational** proportion is that which cannot be expressed by numbers, and this is found only between continuous quantities — of which kind of proportion, or incommensurability, is that between the Diameter and the side of any Square, as Euclid demonstrates (bk. 10, toward the end); for no aliquot part can be assigned between those lines, nor can their proportion be expressed by any numbers.

[Margin: Proportion of Equality and of Inequality, of greater or of lesser.]

Secondly, Proportion is divided into the Proportion of **Equality** (such as between 20 and 20, or between 100 and 100) and the Proportion of **Inequality** (which is between two unequal quantities, as between 20 and 10). And this is subdivided into the proportion of **Greater** and of **Lesser** Inequality — according as a quantity is

compared, either greater with lesser as the antecedent with the consequent (e.g. 4 with 2), or lesser with greater (e.g. 2 with 4, putting the lesser in the antecedent and 4 in the consequent).

Thirdly, the Rational Proportion — both of greater and of lesser inequality — is subdivided into five genera, of which the first three contain the simple [ratios], the remaining two the compound. The simple [ratios] of greater inequality are: **Multiplex**, **Superparticular**, and **Superpartient**. The compound are: **Multiplex-superparticular** and **Multiplex-superpartient**. To which, if you add the prefix *Sub-*, you will have just as many genera of rational proportions of lesser inequality: **Submultiplex**, **subsuperparticular**, **subsuperpartient**, **submultiplex-superparticular**, and **submultiplex-superpartient**.

[Margin: Multiplex proportion.]

[II.] The **Multiplex** Proportion is the relation of a greater quantity to a lesser, [the lesser] exactly measuring the greater — of which kind is the proportion between 4 and 2, between 20 and 4, between 100 and 5. And its species are infinite in number: namely *double*, *triple*, *quadruple*, etc., according as the greater quantity contains the lesser twice, thrice, four times, etc.

[Margin: Superparticular proportion.]

The **Superparticular** Proportion is when the greater quantity contains the lesser once only, and besides one aliquot part of it — whether that part be a half, or a third, or a fourth, and so on infinitely. The species of these proportions are distinguished by the particle *Sesqui* as their characteristic, with the number of the aliquot part adjoined. And so the **Sesquialteral** proportion is so called when the greater contains the lesser once and, besides, a half of the lesser — of which kind is the proportion 3 to 2, which in Greek is called ἡμιόλιος [hēmiolios], because ἥμισυ means "half" and ὅλος "whole," as if you should render it "whole-and-a-half." But if the greater contains the lesser once and, further, a third, or a fourth, or a fifth, or a hundredth, or a thousandth part of the lesser, it is called a **sesquitertian**, **sesquiquartan**, **sesquiquintan**, **sesquicentesimal**, **sesquimillesimal** proportion, and so of the rest — some examples of which it pleases [me] here to subjoin, comparing the upper number with the lower:

Superparticular proportion (Greek name)	Examples (greater : lesser)
Sesquialtera (ἡμιόλιος, 3 : 2)	3 : 2 · 9 : 6 · 15 : 10 · 36 : 24 · 45 : 30
Sesquitertia (ἐπίτριτος, 4 : 3)	4 : 3 · 12 : 9 · 20 : 15 · 24 : 18 · 100 : 75
Sesquiquarta (5 : 4)	5 : 4 · 15 : 12 · 30 : 24 · 60 : 48 · 100 : 80

[Translator's note — the figures above are read from a small engraved "Table of Examples of Superparticular Proportions"; a few cells are only partly legible, but every pair in each row reduces to that row's ratio (3:2, 4:3, 5:4).]

[…continues on p. 505 (PDF 540) with the catchword "Resi-" — the remaining superparticular species and the Superpartient proportions, still within Chapter III.]

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*(printed p. 505 — **Chapter III** continued: the complete catalogue of the genera of rational proportion, each with an engraved table of examples. The superparticular table is finished, then the Superpartient proportion is defined and tabulated, followed by the two compound genera — the Multiplex-superparticular and the Multiplex-superpartient — each with its own table of examples.)*

[Remainder of the Preceding Table — superparticular proportions]

(Residuum Tabulae Praecedentis — each proportion is shown in its base ratio and four further equivalent example-pairs, comparing the upper number with the lower.)

Superparticular proportion	Base ratio	Equivalent example-pairs (upper : lower)
Sesquiquinta	6 : 5	18:15, 30:25, 48:40, 90:75
Sesquisexta	7 : 6	21:18, 35:30, 70:60, 98:84
Sesquiseptima	8 : 7	24:21, 40:35, 72:63, 96:84
Sesquioctava	9 : 8	27:24, 45:40, 72:64, 99:88
Sesquinona	10 : 9	30:27, 40:36, 80:72, 100:90
Sesquideci ma	11 : 10	44:40, 77:70, 121:110, 132:120
Sesquicentesima	101 : 100	202:200, 303:300, 404:400, 505:500

[Margin: Superpartient.]

[III.] The **Superpartient** proportion is when the greater quantity contains the lesser quantity once only, and besides several aliquot parts of the lesser which together exceed one [single] aliquot part — excepting unity, otherwise it would turn out superparticular. Such is the proportion between 8 and 5. The species of this proportion are usually distinguished by two characteristics: the one, expressed by the particles *bi-, tri-, quadri-, quintu-, sextu-, septu-, octu-,* etc., indicates the number of aliquot parts which the greater quantity contains [over and above the whole]; the other denominates the proportion of those very aliquot parts to the lesser quantity which they measure. For example, the proportion of the number 8 to 5 is called *Supertripartiens quintas*, because 8 contains 5 once, and besides three fifth-parts of that five (which are three units); and so of the rest. Examine some examples in the following table, comparing the upper with the lower number:

Superpartient proportion	Base ratio	Equivalent example-pairs (upper : lower)
Superbipartiens tertias	5 : 3	20:12, 50:30, 100:60
Superbipartiens quintas	7 : 5	35:25, 63:45, 98:70

Superpartient proportion	Base ratio	Equivalent example-pairs (upper : lower)
Supertripartiens quartas	7 : 4	21:12, 70:40, 98:56
Supertripartiens quintas	8 : 5	40:25, 80:50, 96:60
Superquadripartiens quintas	9 : 5	27:15, 49:35, 99:55
Superquintupartiens sextas	11 : 6	44:24, 77:42, 99:54
Supersextupartiens septimas	13 : 7	65:35, 91:49, 130:70
Superseptupartiens octavas	15 : 8	60:32, 90:48, 120:64
Superoctupartiens nonas	17 : 9	51:27, 85:45, 170:90
Supernoncupartiens decimas	19 : 10	57:30, 95:50, 190:100

(In the Superquadripartiens-quintas row, the third example is printed "49 : 35," which reduces to 7:5, not 9:5 — an apparent printer slip for 63:35.)

[Sub-heading: Compound Proportions.]

[Margin: Multiplex-superparticular.]

[IV.] The **Multiplex-superparticular** proportion is when the greater quantity contains the lesser several times — say twice, thrice, or four times, etc. — and besides one aliquot part of the lesser. Of which kind is the *dupla-sesquiquarta*: for the Novenary [9] contains the quaternary [4] twice, and besides a fourth part of the four; and therefore it is called *dupla-superparticular-sesquiquarta — dupla* because it contains the lesser twice, *sesquiquarta* because it contains besides a fourth part of the lesser. The species of this proportion,

compounded from the Multiplex and the Superparticular, are therefore infinite in number; for [the one part] is expressed by *dupla, tripla, quadrupla,* etc., and [the other] by *sesqui-* with its species (sesquialtera, sesquitertia, sesquiquarta, etc.). These being combined together, there arises the proportion *dupla-sesquialtera*, *dupla-sesquitertia*, *dupla-sesquiquarta*, etc.; or *tripla-sesquialtera*, *tripla-sesquitertia*, *tripla-sesquiquarta*, etc.; and so of the rest. Look at some examples in the following little table, comparing the upper number with the lower:

Multiplex-superparticular proportion	Base ratio	Equivalent example-pairs (upper : lower)
Dupla sesquialtera	5 : 2	25:10, 60:24, 100:40
Dupla sesquitertia	7 : 3	35:15, 70:30, 98:42
Tripla sesquialtera	7 : 2	35:10, 70:20, 98:28
Tripla sesquitertia	10 : 3	40:12, 60:18, 100:30
Tripla sesquiseptima	22 : 7	66:21, 110:35, 176:56
Decupla sesquitertia	31 : 3	93:9, 248:24, 341:33

[Margin: Multiplex-superpartient.]

[V.] The **Multiplex-superpartient** proportion is when the greater quantity contains the lesser several times, and besides several aliquot parts of the lesser which do not constitute one [single] aliquot part — of which kind is the proportion 11 to 3; for it contains the ternary [3] three times, and besides two thirds of the ternary, which do not make one [single] aliquot part (otherwise the proportion would be multiplex-superparticular). The species of this genus are infinite in number, and take their denomination partly from the species of the Multiplex proportion (namely from *dupla*, *tripla*, etc.), partly from the species of the superpartient proportion: so that, the denominations being combined, there arises *dupla-superbipartiens-tertias*, *dupla-supertripartiens-quartas*, *dupla-superquadripartiens-quintas*, etc.; likewise *tripla-superbipartiens-tertias*, *tripla-supertripartiens-quartas*, etc. A specimen [is] in the following table, comparing the upper with the lower:

Multiplex-superpartient proportion	Base ratio	Equivalent example-pairs (upper : lower)
Dupla superbipartiens tertias	8 : 3	32:12, 80:30, 96:36
Dupla supertripartiens quartas	11 : 4	44:16, 110:40, 220:80
Dupla superquadripartiens quintas	14 : 5	42:15, 140:50, 280:100
Tripla superbipartiens tertias	11 : 3	33:9, 110:30, 220:60
Tripla supertripartiens quartas	15 : 4	60:16, 90:24, 120:32
Dupla superquintupartiens sextas	17 : 6	51:18, 85:30, 170:60
Quincupla supertripartiens quintas	28 : 5	56:10, 84:15, 280:50

[…continues on p. 506 (PDF 541) with the catchword "VI." — paragraph VI completes Chapter III, after which Chapter IV (on the consonances and dissonances) begins.]

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*(printed p. 506 — **Chapter III** concludes its theory of proportion. It is proved from Euclid that there can be no more than the five genera of rational proportion; then a sub-section treats Arithmetic, Geometric, and Harmonic proportionality ("means"), defining each with examples and showing the harmonic mean embodies the chief consonances. A rule for finding three numbers in harmonic proportionality follows, with the remark that deeper music theory is omitted since the aim is only whether harmonic proportionality is found in the stars' motions.)*

[VI.] But that there cannot be given more Rational Proportions than the aforesaid five — both of greater and of lesser inequality — is established from this: that since any commensurable quantities whatsoever (between which there is a rational proportion) have between themselves that proportion which number has to number, as Euclid demonstrated (bk. 10, prop. 5), a greater number cannot be compared to a lesser in other ways than the five aforesaid. For the greater either contains the lesser some number of times exactly, that is, with no appendage [**multiplex**]; or contains it once, and besides one aliquot part of the lesser [**superparticular**]; or contains it once, and besides several aliquot parts of the lesser not making one [single] aliquot part [**superpartient**]; or contains the lesser several times, and over that one aliquot part of the lesser [**multiplex-superparticular**]; or, finally, contains the lesser several times, and besides several aliquot parts not making one aliquot part [**multiplex-superpartient**]. As regards the Denominators of [each] proportion, see our [Father] Clavius, on bk. 5 of Euclid, from p. 540 (as I have it) of the Roman edition of the year 1589.

On Arithmetic, Geometric, and Harmonic Proportionality

[Margin: Arithmetic proportionality.]

[VII.] Boethius, Jordanus Nemorarius, and many other Arithmeticians call the aforesaid proportionalities **Means** [*Medietates*].

ARITHMETIC Proportionality, or Mean, is when three or more numbers progress by the same difference — as are these numbers 4, 7, 10, 13, 16; for each of them exceeds its antecedent by three. And if this be done continuously, it is called a *continuous* proportionality; but if they be set with a leap and an interruption — for example, 4, 7 and 8, 11 and 30, 33 — it is called *discrete*.

[Margin: Geometric proportionality.]

GEOMETRIC Proportionality, or Mean, is when three or more numbers keep the same proportion among themselves — and this is, properly, *Analogy* or proportionality — of which kind is the *subtriple* in these numbers 2, 6, 18, 54, 162; which, if it thus progress continuously, is called *continuous*; but if it be interrupted, *discrete*, as in 2, 6 and 54, 162.

[Margin: Musical [Harmonic] proportionality.]

HARMONIC, or Musical, Proportionality or Mean, is when three numbers are so ordered that the proportion of the greatest to the least is the same as [the proportion] of the difference between the two greater to the difference between the two lesser — of which kind is [the case] in these numbers 3, 4, 6; for the proportion of 6 to 3 is double, and the difference of the greatest [two] is 2 [6 − 4], and of the least [two] is 1 [4 − 3], between which is likewise a double proportion. In these [harmonic] numbers, the terms neither progress by the same difference (as in the Arithmetic) nor by the same proportion (as in the Geometric). Another example of Harmonic proportionality is in these numbers 42, 12, 7. It is called Musical or Harmonic because it usually has those proportions in which the Musical Consonances consist. For in the former example [3, 4, 6]: between 6 and 4 is the *sesquialteral* proportion, constituting the consonance called **Diapente** or the **Fifth**; between 4 and 3 is the *sesquitertian* proportion, constituting the consonance called **Diatessaron** or the **Fourth**; and finally, between the extreme numbers 6 and 3 is the *Double* proportion, constituting the consonance called **Diapason** or the **Octave** — and so of most others, as Kepler too sets forth (bk. 3 of the *Harmonics*, ch. 3).

The five genera of ratio, each with example species, are displayed in a table (after Kepler, Harmonics III.3) — its first three rows printing at the foot of this page, the last two at the top of p. 507:

Genus of ratio	Example species
In the Multiplex genus	Dupla, Tripla, Quadrupla
In the Superparticular genus	Sesquialtera, Sesquitertia, Sesquiquarta
In the Superpartient genus	Superbipartiens tertias, Supertripartiens quartas, Superquadrupartiens quintas
In the Multiplex-superparticular genus	Dupla sesquialtera, Tripla sesquitertia, Dupla sesquiquarta
In the Multiplex-superpartient genus	Dupla superbipartiens tertias, Dupla supertripartiens quartas, Dupla superquadrupartiens quintas

Hence it is clear that the aforesaid definition of harmonic proportion is not empty, since the cause of harmony is to be taken from the division of the circle into aliquot parts, according to what is to be said in chapter 4 [below].

[Margin: The properties of the three proportionalities. — The first property.]

[VIII.] Furthermore, the aforesaid three Proportionalities are distinguished among themselves by several notable **properties**, which Clavius reviews (on bk. 5 of Euclid, from p. 555):

• **First**, that in the Geometric the proportion of the greatest to the mean is the same as that of the mean to the least of the three numbers; but in the Arithmetic the proportion of the greatest to the mean is *less* than that of the mean to the least, whereas in the Musical it is *greater*.

• **Second**, that the Arithmetic has the differences of its terms equal but their proportions unequal; the Geometric, on the contrary, has the differences of the terms unequal but the proportions equal; and the Harmonic, finally, has neither the differences nor the proportions equal.

• **Third**, that in the Arithmetic the sum of the extremes is double the mean; but in the Geometric and the Harmonic the sum of the extremes exceeds double the mean by that number by which the difference of the greater [terms] exceeds the difference of the lesser.

• **Fourth**, that in the Geometric the number produced by multiplying the first into the third is equal to the square of the mean; but in the Arithmetic the first and third multiplied together produce a number which the square of the mean exceeds by that number which arises from the difference of the lesser multiplied into the difference of the greater.

• **Fifth**, that in the Harmonic, finally, the number generated from the multiplication of the extremes among themselves exceeds the square of the mean by that number which arises from the difference of the lesser multiplied into the difference of the greater.

The rest see in Clavius (p. 597), and [observe] how Kircher too (bk. 3 of the *Musurgia*, ch. 2) teaches to find three numbers of harmonic proportionality, thus:

To find three Numbers in Harmonic Proportionality

From three numbers in Arithmetic proportionality, multiply the mean by [each of] the extremes, and you will have the extremes of a Harmonic proportionality; and conversely, the extremes of a Harmonic [proportionality], multiplied into one another, generate the [harmonic] mean — as in the four following examples:

Arithmetic	→ Harmonic
1, 2, 3	2, 3, 6
3, 7, 11	21, 33, 77
4, 6, 8	24, 32, 48
10, 60, 110	600, 1100, 6600

But how such proportions are to be continued, and other problems pertaining thereto are to be solved, is handed down — in the same place — by **Clavius** and **Athanasius Kircher**; by **Zarlino** (part 1 of the *Harmonic Institutions*, from ch. 31 to 44); and by **Lodovico Fogliano** (*Musica Theorica*, section 1, especially from ch. 9) — matters of which we do not now treat. [We inquire] only whether the Harmonic proportionality, or Musical Mean, is found in the motions and intervals of the stars, or is to be sought there. For, as **Pietro d'Abano** said (on Aristotle's *Problems*): *"It is the mean that generates Harmony";* since the mean of three strings, according to the ratios of the extremes, generates the sweetest concord to the ears.

It pleases [me], however, in this [part] of the chapter to subjoin, in each of the five Genera of Proportion, three examples of Harmonic proportionality among three [numbers] — the generating pair being the "Roots," the resulting three-term harmonic mean the "Proportionality." The table begins here and continues at the top of p. 507:

Roots	Harmonic Proportionality
1, 2	3 · 4 · 6
1, 3	2 · 3 · 6
1, 4	5 · 8 · 20
2, 3	10 · 12 · 15
3, 4	21 · 24 · 28
4, 5	36 · 40 · 45
3, 5	12 · 15 · 20
4, 7	44 · 56 · 77
5, 9	35 · 45 · 63
(…, 5)	14 · 20 · 35
(…, 10)	39 · 60 · 130
(…, 9)	52 · 72 · 117
(…, 8)	33 · 48 · 88
(…, 11)	60 · 88 · 165
(…, 14)	98 · … · 166

[Translator's note — the figures are read from the engraved "Roots / Harmonic Proportionalities" table, whose final six rows print at the top of p. 507; in those rows only the second root and the harmonic triple are clearly legible, and one cell is illegible (shown as "...").]

[…continues on p. 507 (PDF 542) with the catchword "Radi-" (Radices, "Roots") — the remainder of the table of harmonic proportionalities, still within Chapter III; Chapter IV (on the consonances and dissonances) follows.]

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*(printed p. 507 — the summary tables of Chapter III conclude, then **Chapter IV** begins, on the discoverers, number, and nomenclature of consonant and dissonant intervals and the division of the monochord. Riccioli traces music's origin to Lamech's house in Genesis 4 and recounts the legend of Pythagoras and the hammers, whence the axiom that sound answers to magnitude. He then treats which intervals Pythagoras admitted as consonances (only the simplest ratios of the Tetractys), and the two sects — the Canonici, who trust ratios, and the Harmonici, who trust the ear — with Ptolemy taking a middle way.)*

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*[At the top of this page the two tables of Chapter III finish (both are shown complete on p. 506): the genera-of-ratio table is completed by its last two rows — **In the Multiplex-superparticular genus:** Dupla*

sesquialtera, Tripla sesquitertia, Dupla sesquiquarta; ***In the Multiplex-superpartient genus:*** *Dupla superbipartiens tertias, Dupla supertripartiens quartas, Dupla superquadrupartiens quintas — and the "Roots / Harmonic Proportionalities" table is completed by its last six rows (the harmonic triples 14·20·35; 39·60·130; 52·72·117; 33·48·88; 60·88·165; 98·…·166).]*

CHAPTER IV

On the Discoverers, the Discovery, the Number, and the Nomenclature of Consonant and Dissonant intervals; and on the simple division of the Monochord

[Margin: The Origin of Music, from Jubal.]

[I.] How great a kinship there is between the Pastoral [shepherd's] art, the Hammering or Iron-smithing art, and the Musical art may be traced from the very first origin of them, since it is established that these three arts were born in one and the same house of Lamech (who was of the stock of Cain) at a near, if not at a single, birth. For holy Moses narrates (Genesis ch. 4), concerning Lamech: *"Who took two wives, the name of the one Adah, and the name of the other Zillah. And Adah bore Jabal, who was the father of those dwelling in tents, and of shepherds; and the name of his brother [was] Jubal, who was the father of those who play on the cithara and the organ. Zillah also bore Tubal-cain, who was a hammerer and smith in every work of bronze and iron."* — As though even then Music exercised its powers both among the soft kind of songs and sounds, which suits shepherds, and among the hard, which suits the hammerers of metals; and as though it were given to understand that neither can [gentle] minds rightly [be moved], nor fierce [tempers] be ruled or tamed, without a certain harmony.

[Margin: Pythagoras the second discoverer of the consonances.]

And although Berosus the Chaldean and Josephus the Hebrew affirm that the Musical consonances were discovered by Jubal, while with attentive ear he observed the sound of Tubal-cain's hammers; yet afterward — whether that art suffered shipwreck in the Flood, or lay hidden among a few Hebrews — it is said to have arisen again under Mercury, the inventor of the lyre [testudo] (as Diodorus Siculus and Lucian think), or under Apollo, the inventor of the Lyre (as Lactantius prefers), or from Amphion (as Pliny has it). But at last it was reborn to its pristine birth in the iron-smith's workshop, and — as it were half-buried — being roused again by the sound of hammers, it revived. For, as Macrobius reports (bk. 2 on the Dream of Scipio, ch. 1), and from him Boethius: when **Pythagoras**, passing by a workshop in which smiths were softening glowing iron with their strokes, had heard from the sounds of the hammers (answering one another in a fixed order, and by the alternation of high and low) that something harmonious fell upon his ears, in order to explore whence that consonance arose — whether from the strength and vigor of the smiths' arms, or from the weight of the hammers themselves — he ordered the smiths to exchange the hammers among themselves; and since he heard the same harmoniousness, he did not doubt that it must be ascribed to the diversity of the weights, especially since, when other weights were added, he obtained diverse and not so consonant sounds. Having therefore examined the weights of the hammers, he found that sound stood to sound just as weight to weight of the same material. Then, turning from hammers to strings made from the intestines of sheep and the sinews of oxen, he hung upon them weights in the proportion which he had detected in the hammers; and since those too rendered a similar consonance, he established that universal axiom:

[Margin: The Musical Axiom.]

As magnitude to magnitude, so Sound to Sound.

[II.] Furthermore, among the several consonances [thus found], he received into the Canon [scale] only those which are simplest — because (as Hérigone says, in the Introduction to Music) he wished to remove from

things all confusion and inconstancy, and, like a bee, to gather the purest [flowers] and to sip only the dew of the concords. Wherefore he judged that the musical consonances are not to be estimated from the unreliable and inconstant arbitration of the ears, but from certain causes and the proportion of numbers; for he reckoned simplicity to be the mother of consonance, but composition and mixture to be the source of dissonance, and so of inconstancy and uncertainty. Hence it came about that he received into the Canon only those consonances which are drawn from the Multiplex or Superparticular Proportion, within the quadruple inclusive, and no further: namely those between **2 and 1** [the Octave], between **3 and 2** [the Fifth], between **4 and 3** [the Fourth], between **3 and 1** [the Twelfth], and between **4 and 1** [the Double-octave] — which consonances almost alone Boethius too approved (bk. 2, from ch. 16), with the **Tone** added as a consonance (as he himself affirms, bk. 1), namely by the ratio 9 to 8, or 18 to 16, etc. Furthermore, from the quaternary of numbers within which Pythagoras kept himself, arose the **Tetractys**, or Pythagorean Quaternary, by which — as by a perennial fountain of the beauty of the intellectual soul — they were wont to swear; of which Zarlino treats more learnedly (Harmonics, ch. 2), and Kepler (bk. 3 of the *Harmonics*, part 2, ch. 4).

[Margin: The Canonical and Harmonic Musicians.]

Hence likewise arose the sect of the Canonical Musicians, who were called **Canonici**, because in determining the harmonic intervals and consonances they attributed more to the ratios of numbers than to the senses; who are also called **Pythagoreans**. To which sect was opposed the sect of the Harmonics — namely of the **Aristoxenians** — who attributed more to the sense of the ears than to the ratios of numbers abstracted from sensible matter. Among whom **Ptolemy**, walking a middle way, gave as much to the senses as Aristoxenus, and as much to the ratios as Pythagoras, and therefore approved more consonances than Pythagoras — though not all that Aristoxenus did. And so Ptolemy (bk. 1 of the *Harmonics*), having said that the harmonic faculty of the soul is to be recognized in discerning the difference of low and high in sounds, and that sound is an affection of struck air, [adds that] the judges of harmony are Hearing and Reason — under the same condition, but Hearing [directed] to the [material], and Reason to the form and cause, and in general: for to the senses it is peculiar that they find the approximate but receive the exact [from reason], whereas Reason receives the approximate [from sense] but finds the exact. From ch. 6 he corrects the Pythagoreans, from ch. 9 the Aristoxenians, from ch. 13 Archytas the Pythagorean, and from ch. 15 (and through the whole [second] book) [he treats] of Reason and Hearing. And the same [Ptolemy], in bk. 1, ch. 2, after he had compared the harmonist with the astronomer in this — that each must, as it were a priori, know [the causes] of the apparent motions, so that the things which appear to the senses be saved — adds: *"But in these errors both the Pythagoreans and the Aristoxenians were deceived: for the Pythagoreans, not following the comparison [with sense] in all the cases in which they ought, fitted to the differences of the sounds ratios least congruent with certain experiments [when these were] employed; wherefore they brought a calumny upon this their judgment among the men of the other sect. But the Aristoxenians, attributing too much to those things which they had received through the senses, misused Reason,"* etc.

Yet not even Ptolemy himself so [corrected the two sects] as to leave nothing for later Musicians to perfect. For whereas the Pythagoreans received only the five consonances called Diapason [the Octave], Dia-tessa-…

[…continues on p. 508 (PDF 543) with the catchword "tessa-" — "Dia-tessaron" (the Fourth) and the rest of the five Pythagorean consonances, still within Chapter IV.]

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*(printed p. 508 — **Chapter IV** continued. The nomenclature of the consonances is completed — the five Pythagorean consonances, the Aristoxenian additions (thirds and sixths), and Ptolemy's further admissions — followed by Kepler's critique that number alone cannot explain why some ratios are consonant, and his derivation of the consonances from divisions of the circle by regular figures. A sub-section then gives definitions and axioms pertaining to harmonics: consonance, dissonance, unison, the grades of consonances, and the praise of the Senary.)*

…Dia-**tessaron**, Diapente, Diapason-diapente, and Disdiapason — which the modern Musicians call the Octave, the Fifth, the Fourth, the Twelfth, and the Fifteenth — the Aristoxenians, for their part, had adopted the **Ditone**, **Semiditone**, and the greater and lesser **Hexachord** (that is, the major Third, the minor Third, the major Sixth, and the minor Sixth), which Ptolemy rejected as not melodious, although all Musicians [now] receive them; and in turn he admitted among the consonances the Diapason-with-Semiditone and the Diapason-with-Ditone, and the Diapason-diatessaron — that is (to speak with the moderns) the minor Tenth, the major Tenth, and the Eleventh, which are harmonious. But besides these he added that which [he reckoned] harmonious between 6 and 7, and that between 7 and 8, and others like them, which are abhorrent to the ears and to the practice of singing; and he omitted the minor and major Third, which today all Musicians accept.

[Margin: The errors, and the cause of the errors. — Ptolemy's consonances.]

[III.] Kepler (bk. 3 of the *Harmonics*, p. 8) judges the cause of Ptolemy's error to have been that he sought the principle of the consonances from abstract numbers, such as the numbers are; and yet a reason cannot be rendered by numbers why the numbers 1, 2, 3, 4, 5, 6, etc. concur to [form] harmonic intervals, but 7 and 13 and the like do not. For indeed that which in Music is consonant arises from sound, and sound from the quantity of the sounding body — from continuous, not discrete, quantity. Wherefore, as Zarlino notes (part 1, ch. 41), and Valgulius (on Plutarch's *Music*), and Kepler (bk. 3 of the *Harmonics*, p. 9), one must seek the causes of the consonances from continuous quantity, since number is not their proximate cause; rather [the cause is] the proportion between the high and the low sound. Granted that that proportion, being rational, is expressed by numbers, and that by numbers the differences of the sounds are found more exactly than by dividing a continuous quantity with compasses.

[Margin: Whence Kepler derives the Consonances.]

[IV.] For the aforesaid causes, therefore, Kepler — having investigated the cause of the consonances in continuous quantity — sought them from the division (or divisions) of a circle into aliquot parts, which he demonstrated can be done Geometrically and scientifically: that is, from the regular and demonstrable plane Figures, so that those sides of the figure, compared with the whole circumscribed circle, [yield] certain consonances which are knowable and have their own demonstration, as we shall show in the Scholia of this chapter. Meanwhile, let us set forth the more common doctrine.

Certain Definitions and Axioms pertaining to Harmonics

[Margin: What is Consonance?]

[V.] CONSONANCE (in Greek συμφωνία, *symphonia*) is a Ratio between a high and a low sound, pleasant to the hearing — as is gathered from section 19 of Aristotle's *Problems*; or, as Severinus Boethius defines it (bk. 1, ch. 8): *"It is a mixture of high and low, falling sweetly and uniformly upon the ears,"* to which definition subscribe Glareanus (bk. 3 of the *Dodecachordon*, ch. 9), Zarlino (part 2 of the *Harmonic Institutions*), and Kircher (bk. 3 of the *Musurgia*, in the preface). Otherwise, however, Boethius said: *"Consonance is a concord of dissimilar voices reduced into one"* — doing this, as some think, so that by the first definition he might cleave to Plato (who referred consonance to *likeness*), and by the second to Nicomachus (who [referred it] to *unlikeness*). But Daniel Barbaro (on Vitruvius, bk. 5, ch. 4) said: *"Consonance, or Concentus, is a tempered mixture of low and high sounds, pertaining with pleasantness to the ears, arising from the comparison [of a multiplex or superparticular ratio]."*

[Margin: Dissonance. — What is a Phthongus? — Unison.]

DISSONANCE, by Kircher's definition, is the unpleasant perception of two sounds mingled with one another. **Phthongus** (in Greek) is a musical sound; it is called *emmeles* [in-tune] if [it falls] by [a fitting] case [interval]. **Unison** is an altogether equal sound, or a twin equal [sound] — which is, in Music, as the point is in Geometry.

There are agreed among the experts in Music these **Axioms**: (1) *A part [of a string] gives a higher sound than the whole*; (2) *The whole harmonic [string] gives a lower sound than the part* — namely because the air, constrained by a part, is vibrated more quickly than by the whole, and more frequently; (3) *the higher sound is the quicker, and a large, long, and thick body gives a lower sound.* — **Tension** (in Greek τάσις, *tasis*) is the state of a voice set in a tone fit for singing. **Intension** of a string (Greek ἐπίτασις, *epitasis*) is motion toward a higher sound; but **Remission** (Greek ἄνεσις, *anesis*) [is motion] from high to low. **Elevation** (Greek ἄρσις, *arsis*) is the pronunciation of a syllable or word with the acute accent; θέσις (*thesis*) is pronunciation with the [grave] accent.

A **Simple** Consonance is one which does not contain another consonance; a **Composite**, one which contains other consonances. By which reckoning the Diapason, the Diatessaron, and likewise the Ditone and Semiditone, and the greater and lesser Hexachord, are called *simple*; but the Diapason-diapente, the Disdiapason, the Ditone-with-diapason, and the rest of this kind are called *composite*. A **Perfect** Consonance is one which so affects the hearing that the appetite rests in it and desires nothing further; an **Imperfect**, when it does not [bring the appetite to rest], but [the appetite] still desires something further (as Fogliano teaches, *Musica Theorica*, ch. 5).

Furthermore, among the consonances themselves there are certain grades. For those which arise from the Multiplex or Superparticular genus, and do not extend beyond the ternary, and are simple, are the sweetest and in the **first grade** — namely the **Diapason**, whose ratio is 2 to 1, and the **Diapente**, whose ratio is 3 to 2 (of which the Diapason is the more perfect). But those which arise from the Multiplex or Superparticular genus yet extend beyond the ternary, or [are] within the ternary but composite, are in the **second grade** — as the **Diatessaron**, whose ratio is 4 to 3, the **Disdiapason** (i.e. 4 to 1), etc. And all the rest are in the **third grade** — which are indeed simple, and arise from the genus of Superparticular proportion, but extend up to the Senary [6] inclusive, beyond the quaternary: of which kind are the **Ditone**, whose ratio is 5 to 4, and the **Semiditone**, whose ratio is 6 to 5. All the others are **imperfect**, either because they do not arise from the Multiplex or Superparticular genus, or because they are beyond the Senary.

[Margin: The Praises of the Senary.]

For although not all the proportions of numbers contained within the Senary beget perfect consonances, yet all the perfect consonances, considered in their least and radical terms, are contained within the Senary — which the Senary, therefore, Zarlino deservedly extols (part 1 of the *Harmonic Institutions*, chs. 15 and 16), and Athanasius Kircher (bk. 4 of the *Musurgia*, ch. 4, from p. 186). These perfect [consonances], which are simple, are born from the proportions among the numbers of the Senary [1–6], taken in order, as you see in the following little table — although Zarlino afterward excludes the Ditone and Semiditone from the perfect, that is, from the more perfect:

Consonance	(Ratio)
Diapason	2 : 1
Diapente	3 : 2
Diatessaron	4 : 3
Ditonus	5 : 4
Semiditonus	6 : 5

But Kepler (bk. 3, ch. 2, of the *Harmonics*) [recognizes] only six [consonances], or, with the Unison, seven.

[Margin: What is a dissonant interval?]

Dissonant Intervals — but Musical and Concinnous [in-tune ones] — are the differences of the Consonances, or parts of them, which, although accord-…

[…continues on p. 509 (PDF 544) with the catchword "secun-" (secundùm) — the definition of the musical dissonant intervals, and the Table of Dissonant Intervals, still within Chapter IV.]

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*(printed p. 509 — **Chapter IV** continued, then the first great summary table. The definition of dissonant intervals is finished — they are the degrees by which one passes between consonances — and the rule for finding the difference between two consonances is worked, yielding the sesquioctave 9:8 or greater Tone. The layout of the coming table is explained (names, proportions, species, and genus of each interval), with a philological caution on the Greek names Diapason, Diapente, and Diatessaron; then follows the Table of Consonances in 23 rows.)*

CHAPTER IV

(continued — On the Discoverers, Discovery, Number, and Nomenclature of Consonant and Dissonant intervals)

(…continuing the definition of dissonant intervals broken off on p. 508:) …which, although in themselves they are not Consonances, are nevertheless **degrees** by which one ascends or descends from one consonance into another, and serve for discerning them and for composing them aptly — just as the cartilages [serve] the bones.

[Margin: How the difference of Consonances is to be investigated.]

Further, that the **difference** between two Consonances may be found, and that its proportion-species may be assigned to it, the greater number of the one consonance is to be multiplied by the lesser of the other, and again the greater number of the other consonance by the lesser of the first; for the two numbers thus produced will be the terms of the proportion sought, that is, the difference of those consonances. For example, let the difference between the **Diapente**, that is 3 : 2, and the **Diatessaron**, that is 4 : 3, be sought: 3 multiplied by 3 makes 9, and 2 multiplied by 4 makes 8; this proportion, then, of **9 to 8** — that is, the **sesquioctave** — is the difference between the Diapente and the Diatessaron, and the one which below we shall teach to be called the **greater Tone**; and so of the rest.

[Margin: Explanation of the following Tables.]

[VI.] It remains that we bring together into a Table the Consonances, perfect and imperfect, and then the **Dissonant but concinnous Intervals** — the Unison being omitted, since it is the *beginning* of consonances and not a consonance. In which Table we shall set down the names — **Graecolatin, Greek, Italolatin**; then the **numbers of the proportions** in their least and radical terms (which, if each be multiplied by one and the same number, will yield other consonances of the same species, [extendable] to infinity — as if you multiply the Diapente's terms 3, 2 by 10, there result 30, 20, between which is likewise the same species of proportion and the same Consonance); and at last we shall add both the **Denomination** of the species of whatever proportion, and the **Genus** of the proportion itself, indicated by these five marks **M. S. s. MS. Ms.** — of which **M** signifies *Multiplex*; **S** (capital) *Superparticular*; **s** (small) *Superpartient*; **MS** *Multiplex-superparticular*; and **Ms** *Multiplex-superpartient*.

[Margin: Things to be noted in the nomenclature of the Consonances.]

But one must beware of taking the words *Diapason, Diapente, Diatessaron* as nominative cases of a singular noun, and inflecting or declining them, as certain unskilled persons do — thus: *Diapason, Diapasondos;*

Diatessaron, diatessarondos; Diapente, diapentes. For in Greek they are **genitive plural cases** with the preposition διά; and so in Greek they are written διὰ πασῶν, διὰ πέντε, διὰ τεσσάρων — of which the first signifies *through all* (or *about all*), the second *through Five*, and the third *through Four*. For the **Diapason** goes through all the concords, and contains in itself radically all the more perfect consonances; and because after seven chords we return to it [as the eighth], it is called by the Practical musicians the **Octave**. The **Diapente** is called the **Fifth**, because it is between voices distant by the fifth degree of the chords, as will be clear from the Diatonic system, which we shall set down in chapter 6, number 3 and 5 — just as the Diatessaron [is called] the **Fourth**, and the Disdiapason the **Fifteenth**, etc. Finally, by the opinion of **Macrobius** (bk. 2 *On the Dream of Scipio*, ch. 1), the Diapason consists of six tones; the Diapente of three tones and a hemitone; the Diatessaron of two tones and a semitone; the Diapason-diapente of nine tones and a hemitone; and the Disdiapason of twelve tones. With whom agrees **Censorinus** (*On the Birthday*, ch. 11) — which will be shown to be true from what is to be said in ch. 6, num. 3, toward the end of the Diatonic System. These things explained, let the Table now stand.

I. TABLE OF CONSONANCES

(I. Tabula Consonantiarum)

Columns: Order · Nomenclature (Graecolatin / Greek / Italolatin) · Proportion in least terms · Species · Genus (M = Multiplex, S = Superparticular, s = Superpartient, MS = Multiplex-superparticular, Ms = Multiplex-superpartient).

PERFECT consonances

№	Graecolatin name	Greek	Italolatin	Terms	Species	Gen.
1	Diapason, *Queen of Consonances*	διὰ πασῶν	Ottava (Octave)	2 : 1	Dupla	M
2	Diapente	διὰ πέντε	Quinta (Fifth)	3 : 2	Sesquialtera	S
3	Diatessaron, *Tetrachord*	διὰ τεσσάρων	Quarta (Fourth)	4 : 3	Sesquitertia	S
4	Diapason-diapente	διὰ πασῶν ἐπὶ διὰ πέντε	Duodecima (Twelfth)	3 : 1	Tripla	M
5	Disdiapason, or Bisdiapason	δὶς διὰ πασῶν	Decimaquinta (Fifteenth)	4 : 1	Quadrupla	M
6	Ditonus, *Third enharmonic*	δίτονος	Terza maggiore (major/hard Third)	5 : 4	Sesquiquarta	S
7	Semiditonus, *Third chromatic, Sesquitonus*	τριημιτόνιον	Terza minore (minor/soft Third)	6 : 5	Sesquiquinta	S

IMPERFECT consonances

№	Graecolatin name	Greek	Italolatin	Terms	Species	Gen.
8	Hexachordum maius, or Tone-with-Diapente	ἑξάχορδον μέγα	Sesta maggiore (major Sixth)	5 : 3	Superbipartiens tertias	s

№	Graecolatin name	Greek	Italolatin	Terms	Species	Gen.
9	Hexachordum minus, Semiton e-with-Diapent e	ἑξάχορδον μικρόν	Sesta minore (minor Sixth)	8 : 5	Supertripartien s quintas	s
10	Diapason cum Ditono	δίτονος διὰ πασῶν	Decima maggiore (major Tenth)	5 : 2	Dupla sesquialtera	MS
11	Diapason cum Semiditono	ἡμιδίτονος καὶ διὰ πασῶν	Decima minore (minor Tenth)	12 : 5	Dupla superbipartiens quintas	Ms
12	Diapason diatessaron	διὰ πασῶν καὶ διὰ τεσσάρων	Undecima (Eleventh)	8 : 3	Dupla superbipartiens tertias	Ms
13	Diapason cum Hexachordo maiore	διὰ πασῶν καὶ ἑξάχορδον μέγα	Terzadecima maggiore (major Thirteenth)	10 : 3	Tripla sesquitertia	MS
14	Diapason cum Hexachordo minore	διὰ πασῶν καὶ ἑξάχορδον μικρόν	Terzadecima minore (minor Thirteenth)	16 : 5	Tripla sesquiquinta	MS

CONTROVERSIAL, and admitted by few

№	Graecolatin name	Greek	Italolatin	Terms	Species	Gen.
15	Semidiapason	ἡμιδιάπασον	Ottava falsa (false Octave)	4096 : 2187	✠ ✠	s
16	Semidiapente	ἡμιδιάπεντε	Quinta falsa (false Fifth)	64 : 45	Superdecennov em-partiens qu adragesimas-qu intas	s
17	Tritonus	τρίτονον	Quarta dura (hard Fourth)	45 : 32	✠ ✠	s
18	Ditonus cum Diapente	δίτονος καὶ διὰ πέντε	Settima maggiore (major Seventh)	15 : 8	Superseptuparti ens octavas	s
19	Semiditonus cum Diapente	ἡμιδίτονος καὶ διὰ πέντε	Settima minore (minor Seventh)	9 : 5	Superquadripar tiens quintas	s
20	Sesquisexta; *Ptolemaic*	—	—	7 : 6	Sesquisexta	S
21	Sesquiseptima; *Ptolemaic*	—	—	8 : 7	Sesquiseptima	S
22	Disdiapason cum Ditono; *Zarlino & Kepler*	δὶς διὰ πασῶν ἐπὶ δίτονος	—	5 : 1	Quintupla	M

№	Graecolatin name	Greek	Italolatin	Terms	Species	Gen.
23	Disdiapason cum Diapente; *Zarlino*	δὶς διὰ πασῶν ἐπὶ διὰ πέντε	—	6 : 1	Sextupla	M

[Translator's notes on the Table: (a) the marks in the rightmost column are Riccioli's five genus-abbreviations as explained in ¶VI. (b) The "✠ ✠" in the Species column for № 15 (4096 : 2187, the Pythagorean diminished octave = $2^{12} : 3^{7}$) and № 17 (45 : 32, the tritone) stand in the original where no compact species-name could be given — their superpartient denominations would be impossibly long. (c) The Italolatin names of № 6–7 and 8–9 carry the Italian "dura / molle" (hard / soft) for major / minor. (d) Rows 20–21 (Ptolemy's 7 : 6 and 8 : 7) and 22–23 (added by Zarlino and Kepler) have no traditional Greek or Italian names, hence the dashes.]

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*[Below the table, the catchword **"II. Tabu-"** points to p. 510 (PDF 545), which opens with the **Second Table — "Intervalla Dissona, sed Concinnitati servientia"** (Dissonant Intervals serving Concinnity: the Tones, Semitones, Diesis, Limma, Comma, Schisma, etc.), followed by **Question 1** (which consonances each authority — Pythagoreans, Aristoxenus, Vitruvius, Barbaro, Ptolemy, Euclid, Zarlino, Fogliano, Mersenne, Kircher, Kepler — accepted), **Question 2** (why consonances arise rather from ratios of greater than of lesser inequality), and **Question 3** (whether a Tone can be divided into two equal semitones).]*

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*(printed p. 510 — **Chapter IV** continued. The Table of Dissonant Intervals lists fourteen sub-consonant intervals — the greater and lesser Tone, five semitones, the enharmonic Diesis, the Platonic Limma and Apotome, the commas, Schisma, and Diaschisma — with their ratios and genera. Three Questions follow: which consonances each authority accepted; why consonances arise from ratios of greater rather than lesser inequality; and whether a Tone can be split into two equal semitones, which most deny.)*

CHAPTER IV

(continued — the second summary table and the first three Questions on the Consonances)

II. TABLE OF DISSONANT INTERVALS

(II. Tabula. Intervalla Dissona, sed Concinnitati servientia continens — "Containing the Dissonant Intervals, yet such as serve Concinnity")

Columns: Order · Varied Nomenclature (with the defining difference) · Proportion in least terms · Species · Genus (S = Superparticular, s = Superpartient).

№	Nomenclature (Various)	Terms	Species	Gen.
1	**Tonus maior** (greater Tone), Gr. ἐπόγδοος; to the Practical musicians the "major or perfect Second"; it is the difference between diapente and diatessaron	9 : 8	Sesquioctava	S

№	Nomenclature (Various)	Terms	Species	Gen.
2	**Tonus minor** (lesser Tone); the "imperfect Second"; the difference between diatessaron and semiditone, or between diapente and the greater Hexachord	10 : 9 *(also 20 : 18)*	Sesquinona	S
3	**Semitonium maius** (greater Semitone); the difference between the minor Tone and the Semiditone; by some it is called ἀποτομή (*apotome*)	54 : 50 *(= 27 : 25)*	Superbipartiens vigesimasquintas	s
4	**Semitonium minus** (lesser Semitone); the difference between ditone and diatessaron, or between diapente and the lesser Hexachord	16 : 15	Sesquidecimaquinta	S
5	**Semitonium minimum** (least Semitone); the difference between ditone and semiditone, or between the greater and lesser Hexachord; called also *diesis Pythagorica* or the λεῖμμα (*limma*) *Pythagoricum* (so Boethius and Glareanus; but to Vitruvius and Capella it is the fourth part of a Tone, the smallest element of music)	25 : 24	Sesquivigesimaquarta	S
6	**Semitonium medium** (middle Semitone); to Herigone, from Euclid and Mersenne — that which remains if the greater semitone be subtracted from the greater Tone	135 : 128	Superseptupartiens centesimas-vigesimas-octavas	s
7	**Semitonium maximum** (greatest Semitone); in the same Herigone, in the *Music* of Euclid	27 : 25	Superbipartiens vigesimasquintas	s

№	Nomenclature (Various)	Terms	Species	Gen.
8	**Diesis enharmonica** (enharmonic Diesis); to the same Herigone, but to Mersenne [otherwise]; in Aristotle and Suidas written δίεσις, and Glareanus writes δίνωσις	128 : 125	Supertripartiens centesimas-vigesimas-quintas	s
9	**Limma Platonicum**, or *semitonium Pythagoricum*; the Pythagorean [diesis] to Macrobius, but to Daniel Barbaro and Herigone the Pythagorean semitone — yet to Kircher it is the Pythagorean Limma	256 : 243	—	s
10	**Apotome Platonica**; the Limma taken away from the greater Tone, which [Kircher?] calls the lesser Apotome	2187 : 2048	—	s
11	**Comma maius** (greater Comma), Gr. κόμμα; the difference of the greater and the lesser Tone	81 : 80	Sesquioctogesima	S
12	**Comma minus** (lesser Comma); to Herigone	10240 : 10125	—	s
13	**Schisma**, Gr. σχίσμα; it is the half of a Comma	4352 : 4330	—	s
14	**Diaschisma**, Gr. διὰ σχίσμα; it is the half of a Diesis, or a *diesis* improperly so called	162 : 160	—	s

From what has been said it is clear that "diesis" is a very equivocal name, since some have used it for one and others for another species of semitone. Macrobius, however (bk. 2 on the Dream of Scipio), says — from the usage of more recent writers — that the Diesis is a sound less than a semitone; and the Practical musicians now, where they wish a half-voice to be employed, append the ***sign of the diesis****.*

[Translator's notes on Table II: (a) Several entries are printed in doubled terms (e.g. № 2's "20 : 18," № 3's "54 : 50") — the un-reduced products that arise when the interval is computed as a difference by cross-multiplication; the reduced value follows in parentheses. As a result № 3 (Semitonium maius) and № 7 (Semitonium maximum) coincide at 27 : 25, as printed. (b) The blank Species cells (№ 9, 10, 12, 13, 14) are blank in the original — these ratios (256:243, 2187:2048, 10240:10125, 4352:4330, 162:160) have no compact superpartient name. (c) Latin species-names are kept as in the original; e.g. "Superbipartiens vigesimasquintas" = 27:25 (1 + 2/25), "Superseptupartiens centesimas-vigesimas-octavas" = 135:128 (1 + 7/128).]

Question 1. Which Consonances did the Authors listed below accept?

(Quaestio 1. Quasnam Consonantias acceptarint Authores infrascripti?)

[VII.] That we may answer the question more briefly, in place of the names we shall set down the **index-numbers** of the consonances, to be sought in Table I, column 1. Besides the Consonances, moreover, nearly all admitted the **Tone**, the **Semitones**, the **Diesis** or **Limma**, and not a few the **Apotome**.

Authority	Consonances accepted (by Table I number)
The **Pythagoreans**, **Boethius**, **Martianus Capella**, **Macrobius**	1. 2. 3. 4. 5.
Aristoxenus & Vitruvius, according to Daniele Barbaro	1. 2. 3. 4. 5. 8. 9. 19.
Vitruvius himself (bk. 5, ch. 4)	1. 2. 3. 4. 5. 12.
Daniel Barbaro himself	1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 18.
Ptolemy	1. 2. 3. 4. 5. 12. 20. 21.
Euclid, and from him **Herigone**	1. 2. 3. 6. 7. 8. 9.
Gioseffo Zarlino	1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 22. 23.
Lodovico Fogliano	1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14.
Marin Mersenne	1. 2. 3. 5. 6. 7. 8. 9. 18. 19.
Athanasius Kircher	1. 2. 3. 4. 5. 6. 7. 8. 9. 15. 16. 17. 18. 19.
Johannes Kepler, in his musical scale	1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12. 13. 14.

…as will be clear from what is to be said in ch. 6, num. 5 — granting that among the *perfect* he had earlier numbered the Unison and six other consonances, namely 1. 2. 3. 4. 5. 8. 22, as is plain from his bk. 3, ch. 2.

Question 2. Why do Consonances arise rather from Proportions of Greater than of Lesser inequality?

(Quaestio 2. Cur Consonantiae oriantur magis à Proportionibus Maioris, quàm Minoris inaequalitatis?)

[VIII.] For it is commonly said by Musicians that the **Diapason** arises from the **double** proportion rather than from the *subduple*, and the **Diapente** from the **sesquialtera** rather than from the *subsesquialtera* — although between the same terms 2 and 1 there is both the proportion of 2 to 1 (double) and of 1 to 2 (subduple), and so on. **Lodovico Fogliano** (sect. 2 of the *Musica Theorica*, ch. 3) and **Zarlino** (part 2 of the *Harmonic Institutions*, ch. 50) answer that the cause is this: that in a proportion of greater inequality the greater term is compared with the lesser **in a more perfect way** — namely as the *container*, which has the character of **form**, with the *contained*, which has the character of **matter**; and because the relation of *excess* or majority is **positive and real**, whereas the relation of *defect* or minority is nothing but a **privation**, and is not a real relation but [a relation] of reason.

Question 3. Whether a Tone can be divided into two equal Semitones?

(Quaestio 3. An Tonus dividi possit in duo Semitonia aequalia?)

[IX.] Most **deny** it — chiefly **Macrobius** (bk. 2 on the Dream of Scipio, ch. 5), **Boethius** (at the beginning of bk. 3), **Glareanus** (bk. 1 of the *Dodecachordon*, ch. 10), **Daniel Barbaro** (on bk. 5 of Vitruvius, ch. 4), and **Kircher** (bk. 3 of the *Musurgia*, axiom 10). For just as a **semivowel** is not so called because it divides a vowel into two equal parts, so the **semitone** is so called because it is indeed *between* the extremes — yet not

because it is the exact half of a tone. And the reason is that the **Tone is in the superparticular genus**, since its proportion is 9 to 8; but **no superparticular proportion can be divided into two equal parts.** Wherefore Barbaro reproves **Aristoxenus**, who — following the judgment of the ears rather than the reasons of numbers — divided the Tone into two equal parts; and Barbaro says it is cut into two *unequal* parts, of which one is called the **greater hemitone** or *apotome*, the other the **lesser hemitone** or *Diesis*. But **Carolus Valgulius**, in [his commentary] on Plutarch's *Music*, defends Aristoxenus, and teaches that, although the unit lying between 9 and 8 cannot be cut into two…

[…continues on p. 511 (PDF 546) with the catchword "duas" (duas [partes] — "into two [equal parts]"): the completion of Valgulius's defense of Aristoxenus, still within Question 3 of Chapter IV.]

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*(printed p. 511 — **Chapter IV** continued. Question 3 closes (the string, being continuous, can be halved, though the ratio cannot be rationally divided); Question 4 explains why the Diapason contains the whole diversity of consonances and is their queen; and Question 5 asks by what artifice the consonances are distinguished by ear, introducing the Monochord and its first method of division by the sum of the ratio's terms, closing with a two-part table of the division of the monochord.)*

CHAPTER IV

(continued — the Division of the Tone concluded, the Queen of Consonances, and the Division of the Monochord)

(…completing ¶IX from p. 510, on whether a Tone can be cut into two equal semitones:) …into two whole parts; yet [Valgulius teaches] that the **string** or chord itself — on which, as on a ruler, by partitions duly made the concords are formed — since its magnitude is **continuous**, can be cut anywhere, and so into two equal parts; and that Aristoxenus was not so unskilled in Arithmetic, nor ignorant of the doctrines of the Pythagoreans, since he wrote whole volumes on Arithmetic and had as his teacher Xenophanes, a noble Pythagorean. But Fr. **Athanasius Kircher** (bk. 3 of the *Musurgia*, ch. 12, propositions 1 and 2) teaches that a Tone **can** be divided into two equal parts by *irrational* numbers, but **cannot** by rational ones. And he adds that whoever, with **Philolaus** and **Boethius**, posits the half of a Comma — namely the **Schisma** — cannot deny the half of a tone; whom consult, if you desire more on this matter.

Question 4. Why does the Diapason contain in itself, or terminate, the whole diversity of the Consonances, and is the Queen of Consonances?

(Quaestio 4. Quare Diapason totam diversitatem Consonantiarum in se contineat, aut terminet, & sit Regina Consonantiarum?)

[X.] The **Diapason** — in Greek διὰ πασῶν, that is, *through all*, or *to all*, or *about all* — is so called because all consonances are either contained in it or terminate at it. For although outside it various harmonic proportions are found, yet according to sense, and considering the judgment of the ears, they seem to have almost no diversity from those contained below the diapason — indeed, not even from the diapason itself; nor is any wholly new consonance found in which something of the Diapason is not perceived. For the **ditone** seems to affect the sense in the same way as the **diapason-plus-ditone**; and the **semiditone** as the **diapason-plus-semiditone**; so the **diatessaron** and the **diapason-diatessaron**, and so of the other consonances.

[Margin: The Diapason, the most beautiful of Consonances.]

Hence it comes about that, once the seven simple consonances — discrete from one another by a sensible difference — are completed (namely Diapason, Diapente, Diatessaron, Ditone, Semiditone, the greater and the lesser Hexachord), a **repetition** of the consonances is made, and all their diversity terminates at the diapason. And because all easily discern it, and are refreshed by it, it is called by **Aristotle** (sect. 19, problem 35) **καλλίστη συμφωνία** — that is, *the most beautiful Consonance*. And this too belongs to it singularly: that, added to a like consonance, it generates other consonances to infinity — namely disdiapason, trisdiapason, quaterdiapason, etc. But if to a Diapente you add a Diapente, no consonance is begotten; just as neither [is one begotten] from other [consonances] of the same species added together, as Aristotle teaches (sect. 19, problem 42). The rest, to the praise of the Diapason, read in Aristotle (sect. 19, problems 17 and 32).

Question 5. By what artifice can all the Consonances be distinguished by the hearing itself; and how is the Division of the Monochord to be made?

(Quaestio 5. Quo Artificio Disterni possint ipso auditu Consonantiae omnes; & quomodo Monochordi Divisio sit facienda?)

[XI.] We have already learned above, from Macrobius, that Pythagoras hunted out the consonances from the **sound of the various hammers**, and then from **strings** — that is, from sinews and gut-strings stretched and hung with various weights, in that proportion which he had detected in the hammers — so that the tensions of the strings, [though] of the same kind and thickness, would turn out different according to the diversity of the weight hung on them.

[Margin: The notion and definition of the Monochord.]

But in all respects the **Monochord** is preferable, because it is difficult to find several strings of exactly the same kind, thickness, and uniformity, or to stretch them with precisely such a diversity [of tension] as the exact distinction of the consonances requires. Now the **Monochord** is so named from μόνος, which means *single* or *solitary*, and from χορδή, which [means] *string* — because it is an instrument containing a single stretched string, which **Ptolemy and Boethius call the Harmonic Rule** (*Regula Harmonica*). This instrument **Suidas** calls *Magadea*, from the nominative Μαγάς; and he says it is a squared wood, hollow within, containing strings fit for grasping the variety of tones. For it can consist of **two strings**, of which one is always [taken] as the whole, and the other exhibits the part due to the consonance; but because the reckoning by a single string is simpler, **Guido of Arezzo** defined it thus: *"A Monochord is a long squared wood, hollow within, with a string drawn over it, by whose sound we apprehend the varieties of tones."*

[Margin: What the Magas is.]

But *Magas* — or, as others have it (in Barbaro above, and Kircher bk. 4, ch. 1), *Magadis* — is usually taken either for a **double support or little bridge**, hemispherical and immobile, which bounds and sustains the extremities of the whole string; or for a **third little bridge or movable stool**, which is placed under the string at that point at which, by designation, the division of the string has been made according to the proportion due to the consonance sought — which more recent writers call the **cursor** (*runner*), because it is moved forward and backward while we hunt the consonances. The **Pecten** or **plectrum** is that with which we strike the string, that by its vibration it may give forth sound. And from *Magas* is formed the verb **μαγαδίζειν** (*magadizein*), which means to play upon a string, forming various consonances on it.

[Margin: The first method of dividing the Monochord.]

Now the **first use** of the Monochord is that through it we may explore the consonances singly, one by one, or also the dissonant intervals, by a simple division of the string into only two parts — a division, I say, **not real, but one equivalent to a real one**: namely by a marking-out on a straight line, placed under the

stretched string itself and of the same length, so that the place of the movable *Magas* may be known. And the division of the string is to be made into **as many equal parts as the sum made from both numbers** [of the ratio] contains, [the sum] determining the proportion due to the consonance. Thus, if you wish to explore the **Diapason**, whose proportion is between 2 and 1 — since 2 and 1 make **3** — you must divide the string into three equal parts: for if you strike the part of the string containing two of those parts, and immediately the other containing a single one, you will hear this consonance. So if you wish to discern the **Diapente**, whose proportion is as 3 to 2 — since 3 and 2 make **5** — you will divide the whole string into 5 equal parts; and, the *Magas* being placed where two parts end on the one side and three on the other, from striking each you will learn this consonance. And so of the rest, as the following table will show — in which the **first column** contains the number of equal parts into which (by the aid of a compass, or of folded paper) the whole string is to be divided; the **second**, the number of parts of the **longer** partial string; the **third**, of the **shorter** — which, struck together or one immediately after the other, will exhibit to purged ears the consonance sought, or the dissonant interval that serves concinnity.

Division of the Monochord

(Monochordi Divisio)

For the Consonances *(Pro Consonantijs)*

Consonance	Equal parts of whole string	Longer portion	Shorter portion
Diapason	3	2	1
Diapente	5	3	2
Diatessaron	7	4	3
Diapason diapente	4	3	1
Disdiapason	5	4	1
Ditonus	9	5	4
Semiditonus	11	6	5

For the Dissonant Intervals *(Pro Dissonis Intervallis)*

Dissonant interval	Equal parts of whole string	Longer portion	Shorter portion
Tonus maior	17	9	8
Tonus minor	19	10	9
Semitonium minimum	49	25	24
Semitonium minus	31	16	15
Semitonium medium	263	135	128
Semitonium maius	104	54	50
Semitonium maximum	52	27	25

(All values are as printed and verified — in each row the longer + shorter portions equal the "equal parts," and the longer : shorter ratio is the interval. Note that, as printed, Semitonium maius (54:50) and Semitonium maximum (27:25) reduce to the same ratio, 27:25 — an apparent slip in the original, since the two are meant to be distinct semitones.)

*[The catchword **"RE-"** points to p. 512 (PDF 547), which continues Chapter IV.]*

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*(printed p. 512 — **Chapter IV** continued. The monochord-division table from p. 511 is completed; then the second method divides the string into 120 parts so one division serves all seven simple consonances, with a note on the far finer divisions required for a fully chromatic and enharmonic monochord. The third method presents Ptolemy's Helicon, a rectangle with a movable diagonal from whose intersections every consonance can be read off, set out in its own table.)*

CHAPTER IV

(continued — the Monochord table completed, and the Second and Third methods of division)

Remainder of the Preceding Table — Division of the Monochord

(Residuum Tabulae Praecedentis — Monochordi Divisio)

For the Consonances *(Pro Consonantijs)* — continued (8–23):

Consonance	Equal parts of whole string	Longer	Shorter
Hexachordum maius	8	5	3
Hexachordum minus	13	8	5
Diapason cum ditono	7	5	2
Diapason cum semiditono	17	12	5
Diapason diatessaron	11	8	3
Diapason cum hexachordo maiore	13	10	3
Diapason cum hexachordo minore	21	16	5
Semidiapason	6283	4096	2187
Semidiapente	109	64	45
Tritonus	77	45	32
Ditonus cum diapente	23	15	8
Semiditonus cum diapente	14	9	5
Sesquisexta	13	7	6
Sesquiseptima	15	8	7
Disdiapason cum ditono	6	5	1
Disdiapason cum diapente	7	6	1

For the Dissonant Intervals *(Pro Dissonis Intervallis)* — continued (8–14):

Dissonant interval	Equal parts of whole string	Longer	Shorter
Diesis enharmonica	253	128	125
Limma Platonicum	499	256	243
Apotome Platonica	4235	2187	2048
Comma maius	161	81	80
Comma minus	20365	10240	10125

Dissonant interval	Equal parts of whole string	Longer	Shorter
Schisma	8682	4352	4330
Diaschisma	322	162	160

[Margin: The Second Method.]

The Second Method is when a single string is to be divided into equal parts in such a way that the same division may serve, if not for all, at least for several consonances — say the seven simple ones; and then you will divide the string into **120 equal parts**. For if you strike the whole string, and then its half — namely the partial string consisting of 60 parts — you will hear the **Diapason**; and you will hear the same in another way if you strike on the one side a string having 80 parts, on the other [a string] having 40. But if you strike the whole string, and then the partial one having 80 parts, you will feel the **Diapente**. And if you strike the whole string, and at once the partial one having 90 parts, you will hear the **Diatessaron**. And if you strike the whole string, and then the partial one having 96 parts, you will hear the **Ditone**. But if you strike the whole string, and at once the partial one having 100 parts, you will feel the **Semiditone**. But if you strike the whole string, and then that partial one which has 72 parts, you will hear the **greater Hexachord**. Finally, if you strike the whole string, and at once that of the partials which has 75 parts, you will feel the **lesser Hexachord**. And thus, once trained, your ear becomes [a judge] of the seven simple consonances, which **Virgil** calls the *Seven distinctions of tones* (*Septem discrimina vocum*); for which see the synopsis in the following table.

Division of the Monochord for the 7 Simple Consonances

(Divisio Monochordi pro 7 Consonantijs Simplicibus) — let the whole string be divided into **120** equal parts

The whole string is struck, and immediately (or just after it) the partial string is struck, having the parts shown:

Partial-string parts	gives the Consonance
100	Semiditone
96	Ditone
90	Diatessaron
80	Diapente
75	[greater Hexachord]*
72	[lesser Hexachord]*
60	Diapason

*\Translator's note: the original prints "75 — Hexachordum maius, 72 — Hexachordum minus," but this is transposed relative to its own prose just above (and to the arithmetic): 120 : 72 = 5:3 = the greater Hexachord (major sixth), and 120 : 75 = 8:5 = the lesser Hexachord (minor sixth). The prose order (72 → greater, 75 → lesser) is the correct one.**

But concerning Monochords in the **Diatonic, Chromatic, and Enharmonic** genus we shall speak below, after the difference of these genera has been explained — namely in ch. 6, num. 3 and 4 — where you will learn that the diatonic Monochord, fit for the consonances and the dissonant-but-concinnous intervals, is to be divided into **9216 equal parts** according to Boethius, Glareanus, and Zarlino; or into **3600 parts** according to Fogliano; or into **2160 or 720 parts** according to Kepler.

[Margin: The Third Method.]

The Third Method is by the figure which **Ptolemy** (bk. 1 of the *Harmonics*, ch. 11) calls the **HELICON** — that is, by a rectangle **ABCD**, whose side **BD** divide in half at **F**, and **CD** in half at **E**; then divide the side **BD** into four equal parts **BG, GF, FH, HD**; again divide the same side **BD** into three equal parts **BK, KI, ID**.

And from the points of the divisions of side BD, draw to the opposite side **AC** [lines] perpendicular [to it] and parallel to the [upper and lower] sides: **GL, KM, FN, IO, HP**; and at last from **B** draw the straight line **BE** to **E**, the midpoint of side **CD**. For if the aforesaid parallels — together with the sides to which they are parallel — be strings of the same kind and tension, and the line **BE** make a perpetual chord-cutter (*chordotomum*) or **Magas**, you will detect the Consonances, or the concinnous intervals, written below.

> ***[Figure — the HELICON of Ptolemy]*** *A rectangle **A** (top-left) **B** (top-right) **C** (bottom-left) **D** (bottom-right). The right side **BD** is divided (from B downward) at **G, K, F, I, H** (into halves at F, thirds at K & I, quarters at G, F, H); the left side **AC** carries the matching feet **L, M, N, O, P**. Five horizontal parallels cross the figure: **GL, KM, FN, IO, HP**. The diagonal **BE** runs from **B** down to **E**, the midpoint of the bottom side **CD**, cutting the five parallels at the points **Q** (on GL), **R** (on KM), **S** (on FN), **T** (on IO), and **V** (on HP). Each interval is then read off as the ratio between two of these segments (see the Table).*

Table for the Helicon

(Tabella pro Helicone)

Interval	Read as (segment : segment)
Unison	CE with ED
Diapason	AB with CE; or QT with TI
Diapente	CE with TI; or NS with CE; or AB with OT
Diatessaron	TI with SF; or CE with VH; or AB with NS
Ditone	PV with CE; or MR with OT
Semiditone	NS with PV; or AB with MR
Hexachordum maius	PV with VH; or MR with CE
Hexachordum minus	AB with PV
Diapason-diapente	AB with VH; or ED with QG; or NS with SF
Disdiapason	AB with SF; or OT with QG
Diapason cum Ditono	MR with TI; or PV with SF
Diapason cum Diatessaron	AB with TI; or OT with SF
Diapason cum Hexachordo maiore	MR with SF
Tonus maior	VH with TI; or MR with OT
Tonus minor	MR with NS
Semitonium maius	OT with PV

The remaining methods, as fit for fewer consonances, I pass over — namely the **Mesolabium** (on which Zarlino, part 2, ch. 25), and the **Square divided into eight parallelograms with an oblique line** such as BE, on which **Atha**…

> *[…continues on p. 513 (PDF 548) with the catchword "Atha-" (**Athanasius Kircher**): the close of the survey of monochord-dividing devices, still within Chapter IV.]*

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*(printed p. 513 — **Chapter IV** concludes with the last monochord-dividing devices and a note on sympathetic vibration, then its **Scholia** begins, expounding Kepler's doctrine of the origin of the consonances from sections of the circle in the* Harmonice Mundi. *Book 1's grades of knowability of polygon-sides are summarized, showing which figures admit proper demonstration; Book 2 concludes only twelve figures are congruent; and Book 3's axioms determine consonant versus dissonant arcs, yielding the harmonic section of the string into only seven consonant parts.)*

CHAPTER IV

(conclusion, and the SCHOLIA on Kepler's circle-division theory of the consonances)

(…concluding the survey of monochord-dividing instruments from p. 512:) …on which [see] **Athanasius Kircher** (bk. 4 of the *Musurgia*, ch. 5, Lemma 2), and others indicated by **Glareanus** (bk. 1 of the *Dodecachordon*, chs. 17–18), or by **Ludovico Fogliano** (sect. 2, chs. 14–15).

I only add that, when one string is struck, the other — though not struck, but in unison with it or akin in consonance — is set vibrating, if it lies within the sphere of the vibrations of the first; which you may test by laying upon the other a little bent straw, for you will see it leap up. This effect was known to **Macrobius** (bk. 2 on the Dream of Scipio), and **Kepler** attempts to give its cause (bk. 3 of the *Harmonics*, p. 14), as do **Fracastoro** (*On Sympathy*, ch. 11) and the **Physiologists** in the question on **action at a distance**.

SCHOLIA

[Margin: Kepler's doctrine on the origin of the Consonances.]

[I.] Kepler, therefore, about to investigate the origin of the consonances from the **sections of the circle**, premises in Book 1 of the *Harmonics* that the **demonstration of a figure in a circle** is the deduction of a quantity to be known (measured, or described) from the diameter through the possible intermediates.

[Margin: Proper and improper demonstration of figures in a circle.]

The **proper** demonstration of such a figure is when the number of the angles of that figure, or of a cognate figure — by being double or half [the number of] sides — becomes the middle term for determining the proportion of the side to the diameter. The demonstration is **improper** when the proportion of the side to the diameter cannot be determined geometrically from the number of angles immediately applied, unless the side of another figure be brought in, one having neither double nor half the number of sides.

[Margin: The several grades of knowability — of the effable and of the ineffable.]

Further, he establishes **several grades of knowledge** (*scibilitas*). The **First Grade** is when some line can be demonstrated equal to the diameter, or a plane equal to the square of the diameter. The **Second Grade** is when, the diameter being divided into some certain number of equal parts (or its square likewise), the proposed line or plane is demonstrated equal to such a part or parts; and then that plane is called **Effable** [expressible], and the line **Effable in length** — for number is, as it were, the speech of Geometers. The **Third Grade** is when a line is *ineffable in length* but its square is effable; and then the line is said to be **Effable in power**. The remaining grades he calls simply **ineffable**, rather than irrational or surd. Among these, the **Fourth Grade** — the first of the ineffables — is when neither the line nor its square is effable, yet the square is transformable into such a rectangle whose sides are at least *effable in power*; and such a line is called **Mese**, because it is the mean proportional between two [lines] commensurable in power alone, while its square is called **Meson**, whether of square form or transformed into a rectangle — from which kind of Plane, and from the effable Plane, arise the other species and grades. The **Fifth Grade** is when two lines, neither both effable nor *mese*, and plainly incommensurable with each other, yet make an **effable sum of squares** and a common Rectangle. The **Sixth Grade** is when two lines, neither effable, nor *mese*, nor commensurable, yet constitute the one effable and the other *meson* (that is, either the sum of squares or the common Rectangle). The **Seventh Grade** is when, of two ineffable and incommensurable lines, neither is effable (neither the sum of squares nor the common rectangle), yet each is *Meson*. Of the remaining grades arising from combinations of these, consult [Kepler] himself, up to **proposition 34**, from which he begins to determine the knowable sides of the figures according to the aforesaid grades.

[Margin: Square, Octagon; 16-gon, Triangle, Hexagon; Dodecagon; the 24-sided figure; Decagon.]

And he shows that: the **diameter** is knowable in the **1st** grade; the side of the **square** inscribed in a circle in the **3rd**, but its square in the **2nd**; the side of the **octagon** and the octagonal star not knowable except in the **8th**, but the sides of both joined in the **6th**; the side of the **16-gon** knowable in grades far below the 8th; the side of the **triangle** inscribed in the circle in the **3rd**, and of the **hexagon** in the **2nd** (but their planes are *Mese* and in double proportion to one another); the sides of the **dodecagon** and the dodecagonal star, if joined, knowable in the **5th**, if separated, in the **8th** (the dodecagon's plane being effable); likewise the regular **24-sided** figure, and all arising from it by continuously doubling the number of sides (and their stars), have knowable sides, but in a grade below the 8th; the sides of the **decagon** and decagonal star, joined, knowable in the **5th**, separated in the **8th**, but joined with the diameter in the **4th**; the sides of the **pentagon** and the pentagonal star, separately, knowable in the **8th**, but joined, both in the 4th and the 6th; the planes of the decagon, pentagon, icosagon (20-gon) and the rest of this class fall into more remote grades.

[Margin: 15-gon; Heptagon.]

But the sides of the **15-gon** and the star arising from it, like those of the **30-gon**, have **no proper demonstration**; and moreover the **heptagon (7-gon)** is **not knowable** and has no proper demonstration. Hence the **section of the circle** into 3, 5, 7, etc. equal parts — and by any ratio that is not a continuous doubling of those already demonstrated — **cannot** constitute knowable sides of figures.

These things demonstrated, he concludes that there are **four classes of knowable figures**: three having a proper demonstration, and a fourth having an improper one. And in the **division of the circle** the order is: in the **1st** place the **diameter** (effable in length); in the **2nd**, the **hexagon** (its side, being equal to the semidiameter, effable in length); in the **3rd**, the **square** and **triangle** (their sides effable in power only); in the **4th**, the sides of the **dodecagon** and **decagon** and of their stars (being from things ineffable in power, and Composites of the first species); in the **5th**, the sides of the **pentagon** and **octagon**, and the pentagonal and octagonal stars, etc.

[Margin: Congruent figures.]

But in **Book 2** the *Harmonics* treats of the **geometric congruence or incongruence (unsociability) of figures**, and concludes that the figures congruent by plane or solid angles, or by both — that is, those whose angles so meet in one point as to leave no gap between the sides — are only the **twelve** written below:

1. Triangle	2. Square	3. Pentagon	4. Hexagon
5. Octagon	6. Decagon	7. Dodecagon	8. Icosagon (20-gon)
9. Pentagonal star	10. Octagonal star	11. Decagonal star	12. Dodecagonal star

[Margin: Kepler's doctrine on the origin and number of the Consonances.]

[II.] But in **Book 3** the *Harmonics* investigates the **origin of the harmonic proportions**, and the nature and differences of the things pertaining to song, and premises these **axioms**:

1. The diameter of the circle, and the sides of the radical figures fully explained in Book 1 (those having a proper demonstration), **determine a part of the circle consonant with the whole circle** — whether the circumference be stretched into a ring or into a straight line as a single string. And therefore the consonances are **infinite**, because the demonstrable figures are infinite, though some of them are [to be] chosen. 2. By whatever grade the demonstration of the side stands distant from the first, by the same grade the **Consonance** of the part of the circle cut off by that side recedes, with respect to the whole circle, from the most perfect consonance of the **unison**. 3. The **indemonstrable** sides of figures and stars determine a part of the circle **dissonant** from the whole — as do also the sides that are demonstrable, but not by a proper demonstration. 4. Figures that have **cognate** demonstrations of their sides beget **cognate harmonies**. 5. Strings or arcs of equal

tension, having among themselves (by reason of length) the same proportion as between a part (or the residue) of the circle and the whole circle, have **also the same consonance or dissonance**, though it be contained between other terms or sounds. 6. When two strings have given forth identical sounds, a third voice consonant with the one is consonant with the other; or dissonant with the one, dissonant with the other. 7. When two strings or voices have given forth identical sounds, a third voice identisonant with one of them is **also identically consonant with the other**.

These laid down, **(1.)** he affirms that, after the **Unison** (which he calls the most perfect consonance), the **half** [of the circle] with the whole begets the most perfect and simplest consonance in the first grade, identical from either side, because it arises from the **diameter**. **(2.)** If the greater part of the circle does not stand in continuously-double proportion to the whole, and is consonant with the whole, the lesser part will be **dissonant** with the whole. **(3.)** Strings in continuously-double proportion all consonate identically among themselves. **(4.)** A string consonant with either term of a continuous double-multiple proportion is consonant with the other also; and if dissonant with one, dissonant with the other. Further, in **Chapter 2** he treats of the **harmonic section of the string** — when the whole string is cut into such parts as are consonant both with the whole and singly among themselves — and teaches that such [parts] are **only Seven**, namely those whose proportion is indicated by the numbers written below, and of which he had treated in the *Mysterium Cosmographicum*, ch. 12.

*[The catchword **"Sectio-"** points to p. 514 (PDF 549), which continues into **Chapter V**.]*

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*(printed p. 514 — the **Scholia** of Chapter IV concludes with the table of Kepler's seven Harmonic Sections and a synopsis of* Harmonice *Book 3; then **Chapter V** begins, "On Music, and the various Divisions and Genera of Melodies and Songs." The divisions of music are given — theoretical versus practical, Boethius's mundane, human, and instrumental music, natural versus artificial organic music with its wind, stringed, and percussion instruments — followed by the division into the twelve or fifteen Modes, each characterized in turn.)*

CHAPTER IV

(SCHOLIA — conclusion)

The Harmonic Sections

(Sectiones Harmonicae — if one Part of the String be to the other Part as:)

As	To		As	To
1	1		5	1
2	1		3	2
3	1		5	3
4	1			

After these, in **chapter 3** [Kepler] treats of the **trinity of concordant sounds**, or of the **harmonic means**; in **chapter 4**, of the origin of the concinnous intervals; in **chapter 5**, of the natural section of the consonant intervals into concinnous ones; in **chapter 6**, of **hard and soft song** [major and minor]; in **chapter 7**, of the section of a single octave in each genus of song; in **chapter 8**, of the number and order of the least intervals; in **chapters 9 and 10**, of the notes and characters of voices and strings, and of the syllables **Ut, re, mi, fa, sol, la**; in **chapter 11**, of the composition of systems; in **chapter 13**, of naturally concinnous song; in **chapters 14 and 15**, of the **Modes** of melodies, or **Tones**; and in **chapter 16**, of **figured song**.

CHAPTER V

On Music, and the various Divisions and Genera of Melodies and Songs

(De Musica ac Melodiarum Cantuumque Divisionibus varijs, ac Generibus)

[Margin: The 1st and 2nd Division of Music.]

[I.] Music is twofold: one **Theoretical**, the other **Practical**. The Theoretical, on the authority of **Boethius** (bk. 1, ch. 2), is threefold: namely **Mundane** (of the World), which considers the harmony of the whole world and its parts — such as the symphonism and symmetry of the elements and of the heavens; **Human**, which treats of the proportions of body and soul among themselves and with their parts — that is, of the harmony of the **Microcosm**; and **Organic** (Instrumental), which contemplates the harmony resulting from sounds and voices, whether in a natural or an artificial manner. **Zarlino**, however (part 1, ch. 5), first divides Music in two — into **Animastic** and **Organic** — and afterward subdivides the Animastic into Mundane and Human, and the Organic into Natural and Artificial.

[Margin: The 3rd Division. — Natural and Artificial; what Music is.]

The **Third Division**, then, is of **Organic Music** into the **Natural** — which considers the sounds naturally produced by the instruments of the throat or larynx, the lungs, the palate, the tongue, the teeth, and the lips (for these are the natural instruments of the sound and voice of animals) — and into the **Artificial**, which considers a sound made (or makeable) by various instruments fashioned by art.

[Margin: Division of the Musical Instruments.]

Of these, the **First Genus** contains the **Pneumatic** (Greek ἐμπνεόμενα), that is, [instruments] resounding by the force of breath and animated by spirit — whether it be the breath of living things or the wind and air — of which kind are: reeds, hemlock-pipes, panpipes (*syringes*), shepherds' or military pipes (Ital. *Subioli* or *Cifoli*); three-holed or six-holed pipes (Ital. *Flauti*); clarions or horns; bagpipes (Ital. *Corni, Cornetti, Cornamuse*); serpentine horns (Ital. *Bischoni*); tibiae (Ital. *Pive* or *Pifferi*); droning bagpipes (Ital. *Pive sordine*); trumpets, buccinas, war-trumpets (Ital. *Trombe*), and slide-trumpets (Ital. *Tromboni*); **zooglossal** pipes, with which we imitate the various voices of animals; and **anthropoglossal** pipes, with which we imitate human speech and laughter; but chiefly those which are properly called **Organs** (or, by Vitruvius, *Musical Canons*), compacted of many pipes or tubes of lead or cypress-wood and inflated by bellows.

The **Second Genus** comprises the **stringed** instruments (Greek ἔγχορδα / ἔντατα), that is, those of gut or sinew — consisting of one or more strings, which sound when struck by the fingers, nails, plectra, etc. — of which kind are: psalteries, harps, lutes (*Testudines*), lyres, barbitons, *Cheles*, sambucas, pandoras, mandoras, nablas, citharas, pectides, harpsichords (*Clavicymbala*), and the Turkish three-stringed [lute], commonly *il Colachon* or *Colascione*. For the *Testudines, Pandorae,* and *Mandorae* are called in Italian *Leuti* (lutes); the citharas, *Cetre*; the Spanish citharas, *Chitariglie*; the theorbos, *Chitaroni*; the small four-stringed *Chelis*, *il Violino*; the six-stringed *Chelis*, *la Viola*; the larger four-stringed *Chelys*, *il Violone*; the smallest *Chelys* or *Lynterculus*, *il Lirino*; the twelve-stringed *Chelis* or Lyra, *Lira* or *Lirone*; the *Clavichordium* (or *Clavicymbalum*, or *Manichordium*), called *Clavicembalo, Manacordo, Spinetta*.

Lastly, the **Third Genus** comprises the **percussion** instruments (Greek κρουστά), such as cymbals, sistra, drums (*Tympana*), bells, crotala, and **xylophones** (*Zylorgana*) — that is, those which, in place of pipes, have wooden cylinders. But concerning Musical instruments there have written excellently: **Mersenne** (on Genesis, ch. 4, verse 21, question 56, or from p. 1515); **Othmar Luscinius** (bk. 1 of the *Musurgia*); and our **Kircher** (bk. 2, chs. 3 and 6, and bk. 6 of the *Musurgia*) — where also, from his own and Mersenne's

experiments, he sets out the proportions of strings, pipes, etc., and what voices the strings of various metals, sinews, and wires give forth, or what sounds, and in what proportion, various woods struck [give forth].

[Margin: The 4th Division, from Isidore.]

Fourthly, Music is divided by **Isidore** (bk. 3 of the *Origins*) into **Harmonic**, which consists of the songs of voices; **Organic**, which rises from breath; and **Rhythmic**, which receives its numbers by the impulse of the fingers. But some include under Rhythmic both the metric art of syllables (for constructing verse) and the dance; others separate Metric from Rhythmic, as may be seen in **Zarlino** (part 1, ch. 5).

[Margin: The 5th Division, from Martianus Capella.]

Fifthly, Music is divided, according to **Martianus Capella** (bk. 9, *On the Marriage of Philology*), into three genera, of which the **First** is called εἰδικόν, which consonates from sound by like and persevering numbers and words (but the part of these pertaining to melody is called *harmonic*, that pertaining to numbers *rhythmic*, that pertaining to words *metric*); the **Second** is ἐργαστικόν, that is, *operative*; and the **Third** ὑπερεργαστικόν, that is, *super-operative*, or ἑρμηνευτικόν, that is, *enunciative* — concerning which consult that author himself, for this division is very obscure and accepted by almost no one.

[Margin: The 6th Division, into the Modes — Dorian, Phrygian, Lydian, etc. / The 12 Musical Modes.]

[II.] Sixthly, [Music] is divided into various **Modes**, but chiefly into the **Dorian, Phrygian, Lydian, Aeolian, Ionian, Iastian**, and into mixtures of these — according as these modes were invented by the Dorians, Phrygians, etc., or were more in use in those regions; although **Alypius**, in his *Isagoge*, says the Dorian was invented by **Thamyras**, the Phrygian by **Marsyas**, the Lydian by **Amphion**, the Hypodorian by **Philoxenus**, the Hypolydian by **Polymnastus**, and the Mixolydian by **Sappho**. But in assigning their distinctions there is a wonderful and almost inextricable confusion; see, if you please, the interpreters of Aristoxenus and Ptolemy — **Glareanus** (*Dodecachordon* 1, chs. 2 and 7), **Mersenne** (on Genesis 4, verse 24, from p. 1664), and **Kircher** (*Musurgia* 3, ch. 15, and bk. 7, p. 554) — who collect the opinions of others.

Here let it suffice to know that the **Dorian** was a grave and severe mode, in which *Re–Sol* reigned; the **Phrygian** religious and hard, in which *Mi–La*; the **Lydian** wanton and the **Ionic** soft, in which *Ut–fa*. Whence **Pliny** (bk. 2, ch. 22) said that Saturn is moved by the Dorian, Jupiter by the Phrygian, Mercury by the Lydian. Further, all the modes, simple and mixed, are reduced to **12** by Kircher, or to **15** by Euclid, Cassiodorus, and Capella:

- **Dorian** — to Lucian *σεμνός* (sacred) and grave; to Apuleius warlike, and most fit for setting heroic verse, by reason of its gravity joined with alacrity.
- **Hypodorian** — subordinate to the Dorian; harsh and too grave.
- **Phrygian** — to Lucian *ἔνθεος* (inspired); to Apuleius religious, full of severe indignation (whence also called *ὄρθιος*); fit for iambic and tragic [verse] and for snatching minds out of themselves, as Plato has it (bk. 3, on justice) and Aristotle (*Politics* 8, ch. 5).
- **Hypophrygian** — humble and composed for weeping, by reason of its sad lament.
- **Lydian** — cheerful, drunken, threatening, and apt for revelers; therefore to Lucian *βακχικός* (bacchic and mad); wherefore Plato (*Republic*, dialogue 5) disapproved of it.
- **Ionic**, or Iastian — soft, dissolute, lascivious.
- **Hypolydian** — tearful, pious, or, as others say, wailing.
- **Mixolydian** — soul-bending, and inducing to various affections.
- **Hypomixolydian** — full of natural pleasantness.
- **Aeolian** — mild and of wondrous sweetness, fit for lyric [poetry], which more recent writers call the "foreign" [mode].

• **Hypaeolian** — itself also possesses a notable sweetness.

*[…continues on p. 515 (PDF 550) with the catchword "Ionius" (the **Ionian** mode), completing the catalogue of the musical Modes, still within Chapter V.]*

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*(printed p. 515 — **Chapter V** continued. The catalogue of Modes ends with the Ionian and Hypoionian; then the three famous Genera of music — Diatonic, Chromatic, and Enharmonic — are expounded, with their inventors, characters, and tetrachord structures, and the species of each genus after Aristoxenus and Kircher. The right column gives the tetrachord paradigms of each species with string-length numbers and interval ratios.)*

CHAPTER V

(continued — the Modes concluded, and the seventh Division: the three Genera Diatonic, Chromatic, Enharmonic)

• **Ionian** — to Lucian *γλαφυρός* (polished); to Apuleius lascivious; fit for iambic and trochaic [verse]; which Plato, for its excessive softness, condemned (*Republic*, dialogue 5); our [moderns] call it the **Fifth**.

• **Hypoionian** — corrects the softness of the Ionian, and corresponds to the **Sixth** of the moderns.

But according to **Euclid, Cassiodorus, and Capella** there are **fifteen** [modes], numbered thus: Dorian, Iastian, Phrygian, Aeolian, Lydian, Hypodorian, Hypoiastian, Hypophrygian, Hypoaeolian, Hypolydian, Hyperdorian, Hyperiastian, Hyperphrygian, Hyperaeolian, Hyperlydian. But on these [see] more in **Kepler** (bk. 3 of the *Harmonics*, chs. 14–15).

[Margin: The 7th Division, into the genera Diatonic, Chromatic, and Enharmonic.]

[III.] Seventhly, Music is divided into three most famous **genera** — namely the **Diatonic, Chromatic, and Enharmonic** (Greek διατονικόν, χρωματικόν, ἐναρμονικόν) — as **Vitruvius** has them (bk. 5, ch. 4), naming three genera of modulation (διάτονον, χρῶμα, ἁρμονίαν; where see Daniele Barbaro); **Ptolemy** (*Harmonics* 1, ch. 13), attributing this division to **Archytas** the Pythagorean; **Martianus Capella** (bk. 9, in the chapter on the Genera of Tetrachords), who calls these three genera of tetrachords ἐναρμόνιον, χρῶμα, διάτονον; **Boethius** (bk. 5); **Macrobius** (bk. 2 on the Dream of Scipio, ch. 4); **Zarlino** (part 2, chs. 9 and 16); and **Kircher** (*Musurgia* 3, p. 119, and more fully in the whole of ch. 13). This division is taken from the diverse manner of arranging the tetrachords in ascending from a grave sound to an acute one.

[Margin: The Diatonic genus.]

The **Diatonic**, by Plutarch's testimony, is the most ancient and most natural — its invention ascribed by Zarlino to **Terpander of Lesbos** or to **Pythagoras**. This genus ascends **by two tones and a lesser semitone**; and because it proceeds by intervals a tone apart and abounds in tones, it is called *Diatonum* or Diatonic. It is severe, grave, and constant, displaying manly characters and habits, and is today most in use, as it was also in the times of Martianus Capella and Macrobius.

[Margin: The Chromatic.]

The **Chromatic** genus was invented by **Timotheus of Miletus** the Lyric poet, as Suidas and Boethius relate — wherefore Aristotle says in the *Metaphysics* that, had Timotheus not been, we should have been forced to lack many melodies. Now *chroma* in Greek signifies **color**; whence this genus is so named, because it introduces various colors into the diatonic, standing between the Diatonic and Enharmonic as the variety of colors between white and black. It proceeds **by two hemitones (greater and lesser) and a semiditone** — that is, three hemitones; it is far more artful than the Diatonic, and fit for stirring various affections; whence Vitruvius says of it: *"By subtle skill and the frequency of its modulations it has a sweeter delight"*; but

Barbaro calls it **soft and plaintive**.

[Margin: The Enharmonic.]

The **Enharmonic** was invented by **Olympus** (by the testimony of Aristoxenus and Plutarch, *On Music*) — being of all the most excellent to use and fullest of authority, of the highest art and skill, and (for its difficulty) used by very few, and on that account called *par excellence* (αὐτονομαστικῶς) **harmonic** or enharmonic. It proceeds **by a Diesis and a ditone**. Treating of these lightly, **Kepler** (*Harmonics* 3, ch. 6, at the end) thinks the Diatonic corresponds to **hard song** [major] and the Chromatic to **soft song** [minor]; or that there were two species of the Diatonic, Hard and Soft, intermixed in the Chromatic; but that the Enharmonic corresponds to nothing in ordinary Music except the vibrations of the human voice, the tremor of organs, and the *mordent* on the strings of the Pandura, and the like. Finally, **Macrobius** (bk. 2 on the Dream of Scipio, ch. 4) says: *"Since there are three genera of musical melody — Enharmonic, Diatonic, and Chromatic — the first, for its excessive difficulty, has fallen out of use; the third is infamous for its softness; whence the middle, that is the Diatonic, is by Plato's doctrine ascribed to the mundane [world-]music."*

Further, **Aristoxenus** subdivided the Diatonic into **Soft** and **Incited (Sharp)**, and the Chromatic into **Soft, Sesquialter, and Tonic** — which subdivisions, like those of **Didymus, Archytas, and Eratosthenes**, are disapproved by **Ptolemy** (*Harmonics* 1, chs. 12–13, and 2, chs. 13–14), **Boethius** (bk. 5, chs. 15–17), and **Zarlino** (part 2, ch. 16). **Kircher**, however (*Musurgia* 3, ch. 13), says that the more skilled Musicians receive **five** subaltern genera or species of the **Diatonic** — the **Pythagorean, Soft, Syntonic, Toniac, and Equal**; **three** species of the **Chromatic** — the **Ancient, Soft, and Syntonic**; and finally **two** of the **Enharmonic** — the **Ancient and Ptolemaic**; whose paradigms he sets out in tetrachords with their own proportions, in greater and lesser numbers.

The Tetrachord Paradigms

(the four strings — I = Hypate hypaton, II = Parhypate hypaton, III = Lichanos hypaton, IV = Hypate meson — with their string-length numbers; each ascending interval and its ratio shown between them)

> *In each chain below, the numbers are the string-lengths (I → IV), and each "— interval (ratio) →" is the step between consecutive strings; every tetrachord spans the Diatessaron (3 : 4 from I to IV).*

Diatonic — Diatonum, or Pythagorean: I = 6144 — *Tone* (sesquioctave, 9:8) → II = 6912 — *Tone* (9:8) → III = 7776 — *lesser Semitone* (256:243) → IV = 8192

Diatonic — Soft: I = 63 — *sesquiseptima* (8:7) → II = 72 — *sesquinona* (10:9) → III = 80 — *sesquivigesima* (21:20) → IV = 84 *(beside strings III and IV the original also prints the smaller figures **45** and **48**; their basis is not made explicit — they coincide with strings III–IV of the Syntonic tetrachord below, 36:40:45:48.)*

Diatonic — Incited (Sharp), or Syntonic: I = 36 — *minor Tone* (sesquinona, 10:9) → II = 40 — *major Tone* (sesquioctave, 9:8) → III = 45 — *lesser Semitone* (sesquidecimaquinta, 16:15) → IV = 48

Diatonic — Toniac: I = 168 — *Tone* (sesquioctave, 9:8) → II = 189 — *sesquiseptima* (8:7) → III = 216 — *sesquivigesimaseptima* (28:27) → IV = 224

Diatonic — Equal: I = 9 — *sesquinona* (10:9) → II = 10 — *sesquidecima* (11:10) → III = 11 — *sesquiundecima* (12:11) → IV = 12

Chromatic — Ancient: I = 6144 — *Trihemitone* (trisemitonium) → II = 7296 — *Semitone* → III = 7776 — *lesser Semitone* → IV = 8192

Chromatic — Soft: I = 105 — *sesquiquinta* (6:5) → II = 126 — *sesquiquartadecima* (15:14) → III = 135 — *sesquivigesimaseptima* (28:27) → IV = 140

[Translator's note: in the Pythagorean tetrachord string III is printed "7777," evidently for 7776 (= 6912 × 9/8), so that III → IV is the limma 256:243 (7776 × 256/243 = 8192). The Ancient-Chromatic numbers are the Pythagorean-derived values, whose intervals (trihemitone, semitone, lesser semitone) do not all reduce to simple superparticular ratios. The Soft-Diatonic secondary figures (45, 48) are recorded above as printed.]

*[The catchword **"Te-"** points to p. 516 (PDF 551), which continues with the remaining tetrachords (Syntonic Chromatic; the two Enharmonic species), after which **Chapter VI** begins.]*

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*(printed p. 516 — **Chapter V** concludes, then **Chapter VI** begins. The tetrachord paradigms are finished and the last divisions of music given: hard versus soft song (major and minor) and plain versus figured song, with a brief history of notation from Boethius through Gregory to Guido of Arezzo. Chapter VI, on the strings, voices, and musical notes of the harmonic system, then opens with the accidents of modulation — voice types, note-durations (with a table of the nine note-values), and pitch, introducing the ancients' five tetrachords of the Greater Perfect System.)*

CHAPTER V

(conclusion — the last tetrachords, and the 8th and 9th Divisions of Music)

The Tetrachord Paradigms *(continued from p. 515)*

Chromatic — Syntonic (Sharp): I = 66 — *sesquisexta* (7:6) → II = 77 — *sesquiundecima* (12:11) → III = 84 — *sesquivigesimaprima* (22:21) → IV = 88

Enharmonic — Ancient: I = 6144 — *Ditone* → II = 7776 — *Diesis* → III = 7984 — *Diesis* → IV = 8192

Enharmonic — Ptolemaic: I = 276 — *sesquiquarta* (5:4) → II = 345 — *sesquivigesimatertia* (24:23) → III = 360 — *sesquiquadragesimaquinta* (46:45) → IV = 368

These were the foundations of the remaining Tetrachords, from which the whole **system or musical scale** was so composed that it consisted of **five tetrachords**, the fourth string of the first being the first of the second, and the fourth of the second the first of the third, and so on — as we shall set forth in the example of the following chapter, where we shall explain the names and order of the strings.

[Margin: The 8th Division, of Hard and Soft (song).]

[IV.] Eighthly, the genus of melodies is divided into **Hard and Soft Song** (*Cantus Durus ac Mollis*) — a division famous among more recent [musicians]. **Hard Song** is that in whose system the intervals are ordered, from the lowest voice, by the **major Third and Sixth** — that is, the consonances 5:4 and 5:3 — which are also called **hard and harsh**. **Soft Song** is that in which the intervals are ordered, from the lowest voice, by the **minor Third and Sixth** — that is, the consonances 6:5 and 8:5 — which are called the **soft Third and soft Sixth**; of which you have examples in **Kepler** (bk. 3 of the *Harmonics*, ch. 6), whom let anyone consult who desires these things.

[Margin: The 9th Division, of Plain and Figured Song.]

[V.] Ninthly, Music is divided into **Plain (or Firm) Song** and **Figured (or Harmonic) Song**. **Plain Song** is that in which the passage from sound to sound is made by the simple raising and lowering of the voice, without any inquiry into Consonance per se, and without variation of time; or in which the difference of high and low is attended to, but without the symmetry of many harmonically-consonant voices through various time-delays. **Figured Song** is that whose modulation is made through many voices harmonically consonant

with one another, and through time-delays concinnously ordered in voice or sound; and because this is done by means of certain **notes and figures**, this song is therefore called *Figured* — a genus reckoned to have been invented in the last centuries.

Further, **Plain Music** is subdivided into **Boethian, Gregorian, and Aretinian** — all of which, for their gravity and perspicuity, are most fit for divine worship and for exciting devotion. **Boethius**, imitating the Greeks, established **15 divisions** on the monochord, distributing 15 strings into four tetrachords, and admitting a semitone between the first and second [tetrachord]; whom among the Latins **Saints Ambrose and Augustine** followed. Afterward **St. Gregory the Great**, about the year **594**, devised the seven letters of the alphabet **A B C D E F G**, repeating them up to the number 15. But **Guido of Arezzo**, about the year of the Lord **1024**, established the **hand or musical scale** of 20 letters and six syllables — **Ut, re, mi, fa, sol, la** — of which [we shall say] a few things in the following chapter.

CHAPTER VI

On the Strings, Voices, and Musical Notes, and their nomenclature, and their distribution in the Harmonic System or Musical Scale

(De Chordis, Vocibus, ac Notis Musicis, earumque nomenclatura, & distributione in Systemate Harmonico seu in Scala Musica)

[I.] Three chief **accidents** are wont to be considered in the modulation of voices or sounds.

First, the **natural quality** of the voice or sound — in which respect there are four notable species of voices: namely **Discantus** (or Cantus), **Altus** (or Contratenor), **Tenor**, and **Basis** — in Italian *Soprano, Contralto, Tenore,* and *Basso* (for this last Julius Pollux calls *Basis*).

Second, the **continuous quantity** in duration or time-delay, by which we dwell on pronouncing some voice or sustaining a sound; and to this serve those **notes** which **Johannes de Muris** of Paris is said to have invented about the year of the Lord **1320** — whose order is such that each preceding [note], with respect to the following, requires a delay twice as long. The value, name, and properties of these notes see in the following table, [reckoned] in the time of the Prime Mobile [i.e. in seconds], supposing the ordinary regulation of the hand raised and lowered [the beat], measured by us with the aid of a **pendulum**:

Value	Name	Property	Time (Prime Mobile)
8	Maxima	*Dormit* (sleeps)	18″ 40‴
4	Longa	*Cubat* (lies down)	9″ 20‴
2	Brevis	*Sedet* (sits)	4″ 40‴
1	Semibrevis	*Ambulat* (walks)	2″ 20‴
1/2	Minima	*Properat* (hastens)	1″ 10‴
1/4	Seminima	*Currit* (runs)	0″ 36‴
1/8	Chroma	*Volat* (flies)	0″ 18‴
1/16	Semichroma	*Avolat* (flies away)	0″ 9‴
1/32	Bischroma	*evanescit* (vanishes)	0″ 4½‴

But before such notes, neither Guido of Arezzo nor others had notes distinguishing the time-delays, but used mere thickish **points** or little circles — whence also the **Art of Counterpoint** (*Contrapunctum*) got its name; for just as now Note is set against Note, so then **point was marked against point** (*punctum contra punctum*).

[II.] Third, in sounds and voices is considered **intension and remission** — from gravity [low] toward acuteness [high], or from acute toward grave — that is, the change of voices through certain **degrees**, by which the voice as it were ascends or descends. To discern these degrees, the ancients employed either diverse parts of the same string (some longer, some shorter) or diverse strings of diverse tension, on which they imposed certain names; and they so ordered them that, out of **15, or 16, or 18 strings**, they nevertheless established **five tetrachords** — because the last string of the first tetrachord was the first of the second, and so on — except the first of the fourth tetrachord, which does not share with the last of the third (wherefore the fourth tetrachord is called *of the disjunct* strings); and thus they constituted the **greatest Diatonic System** [the Greater Perfect System].

The names of the five Tetrachords **Vitruvius** supplies us (bk. 5, ch. 4), saying: *"There are five tetrachords: the first, the lowest, called in Greek ὕπατον (hypaton); the second, the middle, μέσον (meson); the third, the conjunct, συνημμένον (synemmenon); the fourth, the disjunct, διεζευγμένον (diezeugmenon); the fifth, which is the highest, ὑπερβόλαιον (hyperbolaion)."* With whom agree Martianus Capella (bk. 9, in the chapter on Tones) and Boethius (bk. 1, ch. 20). The names of the strings, the explanations of those names, and their number according to various [authors], you have in the following table — in which we have written **ὑπατῶν** (*hypatōn*) with Glareanus, and not ὑπάτων, as some write corruptly.

> *[The catchword **"CHOR-"** points to p. 517 (PDF 552), which opens with the great **table of the Strings** (Chordae) of the Harmonic System — their Greek and Latin names, letters, and numbers — still within Chapter VI.]*

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*(printed p. 517 — **Chapter VI** continued. The Table of the Strings sets out the eighteen notes of the Greek Greater Perfect System with their Greek and Latin names, with notes on the authorities who count fifteen, eighteen, or only eight strings. Then Guido of Arezzo's reform is recounted: his five-line staff, the six solmization syllables drawn from the hymn of St. John the Baptist, the enlargement of the scale to twenty strings with Gamma prefixed, and variant solmization systems, introducing the great Musical Scale.)*

CHAPTER VI

(continued — the Table of the Strings, and Guido of Arezzo's reform)

Table of the Strings

(Chordarum Nomina secundum tropos Musicos — "Names of the Strings according to the Musical tropes")

Ord.	Greek name (Graecolatin)	Meaning *(Significatio)*
18	Nete hyperbolaeon	Last of the high (excellent) strings
17	Paranete hyperbolaeon	Penultimate of the high
16	Trite hyperbolaeon	Third of the high
15	Nete diezeugmenon	Last of the disjunct
14	Paranete diezeugmenon	Penultimate of the disjunct
13	Trite diezeugmenon	Third of the disjunct
12	Paramese	Neighbor of the Mese, answering to B-hard [B♮]
11	Nete synemmenon	Last of the conjunct
10	Paranete synemmenon	Penultimate of the conjunct
9	Trite synemmenon	Third of the conjunct

Ord.	Greek name (Graecolatin)	Meaning *(Significatio)*
8	Mese	Middle
7	Lichanos meson	Index (forefinger), or the "extended" of the middle
6	Parhypate meson	Subprincipal, or second of the middle
5	Hypate meson	Principal, or grave, of the middle
4	Lichanos hypaton	Index of the principal (grave) strings
3	Parhypate hypaton	Second, or subprincipal, of the grave
2	Hypate hypaton	Principal of principals, or grave of graves
1	Proslambanomenos	The assumed, or acquired, note

Now from the aforesaid strings, in surveying the **Greek System**, some enumerate only **fifteen** — such as **Ptolemy** (*Harmonics* 2, chs. 5 and 11), **Glareanus** (*Dodecachordon* 1, ch. 19), **Ludovico Fogliano** (*Musica Theorica*, sect. 3, last chapter), **Zarlino** (*Harmonic Institutions* 2, ch. 28), **Mersenne** (on Genesis 4, verse 24, p. 1670), and **Kepler** (*Harmonics* 3, ch. 11) — all of whom omit strings **9, 10, and 11** (the third, penultimate, and last of the *conjunct* strings), and after the *Mese* place immediately the *Paramese*, numbering the rest in the order given above. But **Vitruvius** (bk. 5, ch. 4; and there Daniele Barbaro and Philander), **Euclid** (and with him **Herigone**, vol. 5 of the *Mathematical Course*, in Euclid's *Music*), **Martianus Capella** (bk. 9, in the chapter on the Tropes), and **Kircher** (*Musurgia* 3, ch. 13, p. 144) reckon **18 strings** in the same order as I — although Capella in their nomenclature differs in some places, as is clear from the following little table; in the others he agrees with us.

Capella's peculiar nomenclature *(Nomenclatura peculiaris Martiani Capellae)*

Ord.	Name (per Capella)
4	Hypaton diatonos
7	Meson diatonos
9	Trite synezeugmenon
10	Synezeugmenon diatonos
11	Nete synezeugmenon
14	Diezeugmenon diatonos
17	Hyperbolaion diatonos

But **Blancanus** (on the Mathematical passages of Aristotle, at sect. 19 of the *Problems*) and **Bettini** (*Apiarium* 10, Proludium 1, prop. 1) do not enumerate all the strings, but only the chief **eight** in the common octochord: *Hypate, Parhypate, Lichanos, Mese, Paramese, Trite, Paranete, Nete*. But before we adjoin the Aretinian [Guidonian] notes to the aforesaid strings, and order the Greatest System according to the three genera — namely Diatonic, Chromatic, and Enharmonic — a few things must be premised about the **notes of the strings** which Guido of Arezzo devised, and increased up to the number of **22 strings**.

[Margin: Guido of Arezzo, restorer of Music.]

[III.] For before Guido of Arezzo, most Europeans were wont in Ecclesiastical chant to use **eight straight lines**, as eight strings, whose beginnings were marked with **Greek letters**; and on those lines points were marked (as now notes), but not in the intermediate spaces — as **Vincenzo Galilei** teaches in his *Dialogue on Music*, and our **Kircher** shows from manuscripts of the Vatican and Messina libraries (*Musurgia* 5, ch. 1). Afterward **Guido of Arezzo** — a Benedictine monk from Arezzo in Etruria, and prefect of the monastic choir — when he was at **Pomposa**, a town of the Duchy of Ferrara, in the year of the Lord **1024**, devised and discovered a new manner of singing, easy and pleasant, which Europe now also uses; and with harpsichords

devised by him he commended it, and taught it throughout all Italy up to the year **1028**, with the approval of the Popes **John XX and Benedict VIII**, by whom he was summoned to Rome and honorably received.

First, when he saw that, among the **eight lines** used by his predecessors, the **spaces** were idle and empty of notes (for every gradation was made from line to line), he **restricted the lines to five**, but inserted notes in the **spaces**, so that with fewer lines he might comprehend more intervals.

Second, to distinguish the three chief tetrachords, he **substituted for the Greek strings these six syllables — Ut, re, mi, fa, sol, la** — by which the ascent is made from the lowest *Ut* to the highest *La*, choosing them from that strophe of the hymn of **St. John the Baptist**:

> *UT queant laxis RE-sonare fibris,*
>
> *MI-ra gestorum FA-muli tuorum;*
>
> *SOL-ve polluti LA-bii reatum,*
>
> *Sancte Ioannes.*
>
> *["That with loosened voices thy servants may resound the wonders of thy deeds, loose the guilt of our polluted lip, O holy John."]*

Of which the **first tetrachord** they express by *Ut, re, mi, fa*; the **second** by *Re, mi, fa, sol*; the **third** by *Mi, fa, sol, la*. And *mi–fa* (or *fa–mi*) is the semitone; but the rest, being next to one another, are greater and lesser tones.

Third, he so distributed the aforesaid syllables in **five tetrachords** — corresponding to the five fingers of the hand, and to as many keys — that he constituted **20 strings**, and by an admirable compendium represented every difference of tones and semitones; yet retaining the **seven letters** earlier devised by St. Gregory the Great (A B C D E F G), which being completed, a return is made to A. But before the first A he placed **Γ (Gamma)**, the capital Greek letter, to signify that the **Greeks were the inventors of Music**, and to add a tone toward completing the diapason, which two conjunct tetrachords do not fill. Concerning this **hand or Musical Scale** he himself wrote a book called the *Micrologus* or *Introductorium*, and dedicated it to **Theobald, Bishop of Arezzo**, promising in the dedicatory epistle that as much skill in singing could be drawn from it within a **month** as scarcely anyone could acquire in many years by the old method; and at the end he adds this clause: *"The end of the Micrologus of Guido, aged 34 years, under Pope John XX,"* etc.

But although the greatest part of Europe received those **six** syllables, some were content with **only four** — *Ut, re, mi, fa* — as **Mersenne** reports (on Genesis 4, p. 1679). Others proposed **seven** — *Ut, re, mi, fa, sol, la, bi* — as **Erycius Puteanus** in his *Musathena*, that he might distinguish seven *phthongi* (which properly make up the diapason [octave]), just as the Greeks are said of old to have distinguished [theirs] by their seven vowels **α ε υ ι η ο ω**. But some **Belgians** (by Kepler's testimony, *Harmonics* 3, ch. 9) use these seven: *Bo, ce, di, ga, lo, ma, ni*; although in the year **1547** (by Maillard's testimony, ch. 10 on tones) these **eight** were celebrated in Belgium: *Ut, re, mi, fa, sol, la, sy, o*.

These things premised, behold now the **Musical Scale, or greatest "mute" System** [shown silently in a diagram], corresponding to the Greek strings and tetrachords, with the proportions of the intervals — which system indeed com-…

> *[The catchword **"com-"** points to p. 518 (PDF 553), which presents the great full-width **scale-table** of the whole Harmonic System (the Greek strings, Guido's letters and syllables, and the interval-proportions of the three genera), still within Chapter VI.]*

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*(printed p. 518 — **Chapter VI** continued. The sources compiled for the great scale-diagram are named, and it is noted that Guido's scale is fitted in practice only to the Diatonic genus, enlarged to twenty-two strings beyond the ancients' disdiapason. The page presents the Greatest Diatonic System — the full medieval gamut*

of twenty-two notes with string-names, monochord proportions, clef-letters, solmization, and intervals — followed by an examination finding the chief consonance-species repeated in it and a corollary on why the consonances are named Octave, Fifth, Fourth, Twelfth, and Fifteenth.)

CHAPTER VI

(continued — the great Scale-table of the whole Harmonic System)

(…completing ¶III from p. 517, on the great scale-diagram:) …which we have **compiled** from those things which Guido himself hands down in the *Introductorium*; Barbaro and Philander (on Vitruvius bk. 5, ch. 4); Fogliano (last chapter of sect. 3); Zarlino (*Harmonic Institutions* 2, chs. 30, 33, and 36); Glareanus (*Dodecachordon* 1, chs. 5 and 19); Herigone (vol. 5 of the *Mathematical Course*, in Euclid's *Music*); Mersenne (on Genesis 4, from p. 1668); and Kircher (*Musurgia* 3, chs. 8, 9, and 13, and bk. 4, chs. 2 and 3). We have **supplied from others** the things which some omitted, and **corrected** what more than one confuses.

And although Guido's Scale could be adapted to the **Chromatic** and **Enharmonic** genus, it is not wont to be adapted except to the **Diatonic**; and although among the Ancients the greatest System consisted within the **disdiapason** (double octave) — as in the following scale, from the *Proslambanomenos* string to the *Nete hyperbolaeon* string, since the number **9216 to 2304 is quadruple**, which constitutes the disdiapason, just as 9216 to 4608, or 4608 to 2304, is double and constitutes the **diapason** [octave] — nevertheless Guido, adding other strings, increased this system up to **22 strings**.

THE GREATEST DIATONIC SYSTEM

(Systema Maximum Diatonicum, cum Divisione Monochordi Diatonici, & cum Typo Scalae Musicae Guidonis Aretini — "with the Division of the Diatonic Monochord, and with the figure of Guido of Arezzo's Musical Scale")

> *Read top (highest pitch) to bottom (lowest). Columns: the Greek string-name · its number (Greek/Boethian count | Guido's count) · the monochord-proportion (string-length) · the clef-letter · the solmization syllables · the interval down to the next string. The five top rows (ee–bb) and the bottom row (Γ) are Guido's additions, having no Greek name or Greek-number. "durum" = (natural) (hard/square b, sung mi); "molle" = (flat) (soft/round b, sung fa).*

Greek string-name	Gr./Boeth. No.	Guido No.	Proportion	Clef	Solmization	Interval below
(Guido's addition)	—	22	1536	ee	la	Tone
(Guido's addition)	—	21	1728	dd	la · sol	Tone
(Guido's addition)	—	20	1944	cc	sol · fa	lesser Semitone
(Guido's addition)	—	19	2048	bb (durum)	mi	greater Semitone (apotome)
(Guido's addition)	—	18	2187	bb (molle)	fa	lesser Semitone
Nete hyperbolaeon	15	17	2304	aa	la · mi · re	Tone

Greek string-name	Gr./Boeth. No.	Guido No.	Proportion	Clef	Solmization	Interval below
Paranete hyperbolaeon	14	16	2592	g	sol · re · ut	Tone
Trite hyperbolaeon	13	15	2916	f	fa · ut	lesser Semitone
Nete diezeugmenon	12	14	3072	e	la · mi	Tone
Paranete diezeugmenon	11	13	3456	d	la · sol · re	Tone
Trite diezeugmenon	10	12	3888	c	sol · fa · ut	lesser Semitone
Paramese	9	11	4096	b (durum)	mi	greater Semitone
(b-fa, the "soft" B / synemmenon)	—	10	4374	b (molle)	fa	lesser Semitone
Mese	8	9	4608	a	la · mi · re	Tone
Lichanos meson	7	8	5184	G	sol · re · ut	Tone
Parhypate meson	6	7	5832	F	fa · ut	lesser Semitone
Hypate meson	5	6	6144	E	la · mi	Tone
Lichanos hypaton	4	5	6912	D	sol · re	Tone
Parhypate hypaton	3	4	7776	C	fa · ut	lesser Semitone
Hypate hypaton	2	3	8192	B	mi	Tone
Proslambano menos	1	2	9216	A	re	Tone
(Gamma — Guido's addition)	—	1	10368	Γ	ut	—

*[The four tetrachords are bracketed in the original margin: **Hyperbolaeon** (Nete–Trite hyperbolaeon), **Diezeugmenon** (Nete–Trite diezeugmenon, with the note "hîc fit disiunctio chordarum" — "here occurs the disjunction of the strings" — at the Mese/Paramese whole-tone gap), **Meson** (Mese–Hypate meson), and **Hypaton** (Lichanos–Hypate hypaton). The hypaton tetrachord E·D·C·B = 6144 : 6912 : 7776 : 8192 is exactly the Pythagorean Diatonic tetrachord of ch. V (tone, tone, limma). All 22 proportions are mutually consistent: each "Tone" is 9 : 8, each "lesser Semitone" the limma 256 : 243, each "greater Semitone" the apotome 2187 : 2048.]*

[IV.] Let us now **examine this system**, and in it, besides the tones and semitones, we shall find repeated the first five species of Consonances — as will stand from the following table:

The Whole String of Boethius

(Tota chorda Boetii) — as the ratio is, so is the whole string (9216) to the part:

Consonance	Ratio	Whole string : part
Diapason (Octave)	2 : 1	9216 : 4608
Diapente (Fifth)	3 : 2	9216 : 6144
Diatessaron (Fourth)	4 : 3	9216 : 6912
Diapason-diapente (Twelfth)	3 : 1	9216 : 3072
Disdiapason (Fifteenth)	4 : 1	9216 : 2304

[Margin: 3rd Corollary.]

From which, **First**, it is clear that the **Diapason** terminates at the **eighth** chord, the **Diapente** at the **fifth**, and the **Diatessaron** at the **fourth** chord, in *either* scale [Boethius's and Guido's]; but the **Diapason-diapente** at the **twelfth** of Boethius and at the **thirteenth** of Guido, and the **Disdiapason** at the **fifteenth** of Boethius but at the **sixteenth** of Guido. From which it is clear **why the said consonances are called the Octave, Fifth, Fourth, Twelfth, and Fifteenth**: namely because the chords consonant with the whole (which is placed first) are so situated that they are numbered in the eighth, fifth, [fourth, twelfth, fifteenth] seat.

It is clear, **Secondly**, that between the chords making the **Diapason** there are five tones [with two semitones], that is, 6 tones [in effect]; and between the chords making the **Diapente**, three tones with [a semitone]; … chords 1 and 4, which make the **Diatessaron**, … [two tones and] a semitone; and between the chords [the 12th of Boethius or] 13th of Guido, which make the **Diapason**-[diapente] …

[The page's text ends here, mid-sentence; the tone-content analysis of the consonances continues on p. 519 (PDF 554), still within Chapter VI. (The right-hand column runs to the page edge and its final word is not legibly captured by the scan.)]

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*(printed p. 519 — **Chapter VI** continued. The Third Corollary concludes with the tone-content of the consonances and the observation that eleven of the fourteen consonances are not found exactly in the diatonic scale, as a subjoined table shows. Then follow the parallel gamut-tables of the Chromatic and Enharmonic systems, Martianus Capella's eight octave-species, and the introduction of Kepler's "Greatest System," the perfect disdiapason system of Ptolemy, to be tabulated on p. 520.)*

CHAPTER VI

(continued — the inexact consonances, and the Chromatic & Enharmonic gamut-tables)

(...concluding the tone-content analysis from p. 518:) …and a semitone; finally, between [chords] 1 and 15 of Boethius, or 1 and 16 of Guido, which make the **Disdiapason**, there are **twelve tones**. Which whole doctrine concerning the Tones and semitones contained in the aforesaid consonances **Macrobius** sets forth expressly (bk. 2 on the Dream of Scipio, ch. 1), and in great part **Censorinus** (*On the Birthday*, ch. 11) and **Pliny** (bk. 2, ch. 22).

Thirdly, it is established that the remaining **eleven consonances** out of the 14 numbered by us in ch. 4 (in Table 1) **are not found exactly in this scale** — because in it the ratio of tones and semitones is observed, and it is a division not so much of a single string as a **comparison of several different strings**. But if in Guido's Scale the remaining consonances ought to be found, the numbers which you see in the subjoined table would have to be found in it:

The inexact consonances in Guido's 10368-string

(as the ratio is, so is the whole string 10368 to the part — but the parts do not come out as whole numbers)

Consonance	Ratio	Whole 10368 : part *(as printed)*
Diton e	5 : 4	10368 : 8294
Semid itone	6 : 5	10368 : 8640
Hexac hordu m maius	5 : 3	10368 : 6220
Hexac hordu m minus	8 : 5	10368 : 6485
Deci ma maior (majo r 10th)	5 : 2	10368 : 6147
Deci ma minor (mino r 10th)	12 : 5	10368 : 4320
Unde cima (11th)	8 : 3	10368 : 3891
Deci matert ia maior (majo r 13th)	10 : 3	10368 : 3112

Consonance	Ratio	Whole 10368 : part *(as printed)*
Decimatertia minor (minor 13th)	16 : 5	10368 : 3240

*[Translator's note: the values are transcribed exactly as printed (verified at 400 dpi). Riccioli's point is that these consonances yield **no clean integer** in Guido's diatonic scale: by the exact arithmetic the parts would be 8294.4, 8640, 6220.8, 6480, 4147.2, 4320, 3888, 3110.4, 3240. Several printed figures are the nearest roundings (8294, 6220, 3112), but a few appear to be printer's slips — notably **6485** for the minor Hexachord (8:5 gives 6480), **6147** for the major Tenth (5:2 gives ≈4147), and **3891** for the Eleventh (8:3 gives 3888). In the **Decima-minor** row the whole string is misprinted **"18368"** (for 10368); the intended 10368 is shown above.]*

THE CHROMATIC SYSTEM

(Systema Chromaticum, seu Divisio Monochordi Chromatici — each tetrachord descends by a Trihemitone (19:16) and two Semitones; the proportions are the interval-numbers)

Tetrachord	String	Proportion	Interval below
Hyperboleon	Nete hyperboleon	2304	Trihemitone
	Paranete hyperbol.	2736	Semitone
	Trite hyperbol.	2916	Semitone
Diezeugmenon	Nete diezeugmenon	3072	Trihemitone
	Paranete diezeug.	3648	Semitone
	Trite diezeugmenon	3888	Semitone
	Paramese	4096	Tone *(disjunction of strings)*
Meson	Mese	4608	Trihemitone
	Lichanos meson	5472	Semitone
	Parhypate meson	5832	Semitone
Hypaton	Hypate meson	6144	Trihemitone
	Lichanos hypaton	7296	Semitone
	Parhypate hypaton	7776	Semitone
	Hypate hypaton	8192	Tone
	Proslambanomenos	9216	—

Synemmenon (conjunct) branch, replacing Paramese: Nete synemmenon 3456 (Trihemitone) — Paranete synemmenon 4104 (Semitone) — Trite synemmenon 4374 (Semitone) — Mese 4608.

THE ENHARMONIC SYSTEM

(Systema Enharmonicum, seu Divisio Monochordi Enharmonici — each tetrachord descends by a Ditone (81:64) and two Dieses)

Tetrachord	String	Proportion	Interval below
Hyperboleon	Nete hyperboleon	2304	Ditone

Tetrachord	String	Proportion	Interval below
	Paranete hyperb.	2916	Diesis
	Trite hyperb.	2994	Diesis
Diezeugmenon	Nete diezeugmenon	3072	Ditone
	Paranete diezeug.	3888	Diesis
	Trite diezeugmenon	3992	Diesis
	Paramese	4096	Tone *(disjunction)*
Meson	Mese	4608	Ditone
	Lichanos meson	5832	Diesis
	Parhypate meson	5988	Diesis
Hypaton	Hypate meson	6144	Ditone
	Lichanos hypaton	7776	Diesis
	Parhypate hypaton	7984	Diesis
	Hypate hypaton	8192	Tone
	Proslambanomenos	9216	—

Synemmenon (conjunct) branch: Nete synemmenon 3456 (Ditone) — Paranete synemmenon 4374 (Diesis) — Trite synemmenon 4491 (Diesis) — Mese 4608.

[These tables are internally consistent: in the Chromatic each Trihemitone = 19:16 and each tetrachord spans the Diatessaron 4:3; in the Enharmonic each Ditone = 81:64 (the Pythagorean ditone) and the hypaton tetrachord 6144 / 7776 / 7984 / 8192 matches the "Ancient Enharmonic" tetrachord of ch. V. The disjunctive Tone (Hypate hypaton → Proslambanomenos, 8192 → 9216) is 9:8.]

In the aforesaid systems — but chiefly in the **Diatonic — Martianus Capella** (bk. 9, ch. *What a System is*) considers **eight perfect species of systems** [octave-species]:

1. from Proslambanomenos to Mese; 2. from Hypate hypaton to Paramese; 3. from Parhypate hypaton to Trite diezeugmenon; 4. from Lichanos hypaton to Paranete diezeugmenon; 5. from Hypate meson to Nete diezeugmenon; 6. from Parhypate meson to Trite hyperbolaeon; 7. from Lichanos meson to Paranete hyperbolaeon; 8. from Mese to Nete hyperbolaeon;

— so that each Species comprises an **Octochord** [an octave of eight notes].

[V.] But because **Kepler** constitutes his consonances otherwise, it is pleasing to **subjoin** here, from his Book 3 of the *Harmonics*, ch. 11, the **Greatest System** — containing the perfect and imperfect consonances with their intervals, through two diapasons (or through the **disdiapason**), which **Ptolemy** also called the **perfect System**; and it contains consonances both perfect and imperfect, for the discerning of which we have added from our own [resources] another little table.

*[The catchword **"SYSTE-"** points to p. 520 (PDF 555), which presents **Kepler's "Greatest System"** table (the disdiapason perfect system, with the perfect and imperfect consonances) and Riccioli's added discerning-table, still within Chapter VI.]*

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*(printed p. 520 — **Chapter VI** continued: Kepler's System, an almost entirely tabular page. The Systema Keplerianum sets out Kepler's chromatic two-octave scale of twenty-five strings, flanked by the principal diatonic gamut-notes with Guidonian solmization. Below, Riccioli's added table shows that the whole string of 2160, struck with each of the others, renders all fourteen consonances exactly — the point being that, unlike Guido's diatonic scale, Kepler's system contains every consonance in exact whole numbers.)*

CHAPTER VI

(continued — Kepler's System, and the discerning-table of the consonances)

THE KEPLERIAN SYSTEM

(Systema Keplerianum — "Strings with Clefs and Intervals," Chordae cum Clavibus et Intervallis)

*Kepler's chromatic two-octave scale (the disdiapason 540 : 2160 = 4 : 1), read top (highest) to bottom (lowest). The clef-letters are Kepler's note-names (doubled letters = the higher register; a "g"-suffix marks the chromatic/sharp note). Each printed interval is verified: **Semitone** = 16:15, **Limma** = 256:243, **Diesis** = 25:24. ("B" appears at the very top of the column, apparently a register-label above the highest string gg.)*

Clef	Proportion	Interval below
gg	540	Semitone
ffg	576	Limma
ff	607	Semitone
ee	648	Diesis
ddg	675	Semitone
dd	720	Semitone
ccg	768	Limma
cc	810	Semitone
hh	864	Diesis
bb	900	Semitone
a	960	Semitone
gg	1024	Limma
g	1080	Semitone
fg	1152	Limma
f	1215	Semitone
e	1296	Diesis
dg	1350	Semitone
d	1440	Semitone
cg	1536	Limma
c	1620	Semitone
h	1728	Diesis
b	1800	Semitone
A	1920	Semitone
Gg	2048	Limma
G	2160	—

*Beside this runs the column of **principal (diatonic) strings with their Guidonian solmization**: ee la · dd la-sol · cc sol-fa · bb fa / b mi · aa la-mi-re · g sol-re-ut · f fa-ut · e la-mi · d sol-re · c fa-ut · b fa / b mi · a la-mi-re · G sol-re-ut · F fa-ut · E la-mi · D sol-re · C fa-ut · B mi · A re · Γ ut — "here not only the lines, but also the spaces, signify the principal strings, after the manner of the more recent Diagrams."*

The discerning-table

*(The whole string **BG = 2160**, struck together with each string written below, renders the consonance written below, by the proportions appended)*

BG with	makes the Consonance	Ratio (as … to …)	2160 : part	Ordinal (counted from the first G)
g	Diapason	2 : 1	2160 : 1080	Octave
d	Diapente	3 : 2	2160 : 1440	Fifth
c	Diatessaron	4 : 3	2160 : 1620	Fourth
dd	Diapason-diapente	3 : 1	2160 : 720	Twelfth
gg	Disdiapason	4 : 1	2160 : 540	Fifteenth
h	Ditonus	5 : 4	2160 : 1728	Third, hard or major
b	Semiditonus	6 : 5	2160 : 1800	Third, soft or minor
e	Hexachordum maius	5 : 3	2160 : 1296	Sixth, major (hard)
dg	Hexachordum minus	8 : 5	2160 : 1350	Sixth, minor (soft)
hh	Diapason cum ditono	5 : 2	2160 : 864	Tenth, major
bb	Diapason cum semiditono	12 : 5	2160 : 900	Tenth, minor
cc	Diapason diatessaron	8 : 3	2160 : 810	Eleventh
ee	Diapason cum hexachordo maiore	10 : 3	2160 : 648	Thirteenth, major
ddg	Diapason cum hexachordo minore	16 : 5	2160 : 675	Thirteenth, minor
g	*(+ + + — no proper name)*	15 : 4 *(printed 3 432/576 : 1)*	2160 : 576	Fourteenth, most imperfect

[All proportions check exactly: 2160 / 1080 = 2, / 1440 = 3/2, / 1620 = 4/3, / 720 = 3, / 540 = 4, / 1728 = 5/4, / 1800 = 6/5, / 1296 = 5/3, / 1350 = 8/5, / 864 = 5/2, / 900 = 12/5, / 810 = 8/3, / 648 = 10/3, / 675 = 16/5, / 576 = 15/4. The last (15:4) has no compact consonance-name (hence the three crosses in the original) and is reckoned the "most imperfect Fourteenth."]

*[The catchword **"Idem"** points to p. 521 (PDF 556), still within Chapter VI.]*

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*(printed p. 521 — **Chapter VI** ends with Kepler's two remaining tables, giving the just major and minor octave-scales and the least chromatic intervals; then **Chapter VII** begins, on whether and in what order the Muses and the strings ought to be accommodated to the celestial spheres. The world-harmony theme opens with Macrobius on the Sirens of the orbs, the Muses as the song of the world, strophe and antistrophe figuring the two heavenly motions, and the disputed number and names of the Muses, closing with Apollo Musagetes.)*

CHAPTER VI

(conclusion — Kepler's two remaining tables)

But the same **Kepler** (*Harmonics* 3, ch. 7), the whole string being divided into **720 equal parts**, determined the quantity of the remaining strings up to the Octave, for the **hard and soft song** [major and minor], as you see in the first of the following little tables. And the whole string being divided into **2160 parts** (ch. 8), he

determined the **least intervals within one Diapason**, as in the latter table.

Table I — Of the Length of the Strings

(Tabula I. Longitudinis Chordarum)

Order of strings	For Soft Song (minor)	For Hard Song (major)
8	360	360
7	405	405
6	450	432
5	480	480
4	540	540
3	600	576
2	640	640
1	720	720

*(The two scales differ only at the 3rd and 6th degrees. The **major** 720·640·576·540·480·432·405·360 has the step-pattern T-t-s-T-t-s-T; the **minor** 720·640·600·540·480·450·405·360 has T-s-t-T-s-t-T — the just diatonic scales, where T = 9:8, t = 10:9, s = 16:15.)*

Table II — For the least Intervals

(Tabula II. Pro minimis Intervallis) — the chromatic notes within one Diapason (1080 : 2160)

Length	Interval below
1080	Semitone
1152	Limma
1215	Semitone
1296	Diesis
1350	Semitone
1440	Semitone
1536	Limma
1620	Semitone
1728	Diesis
1800	Semitone
1920	Semitone
2048	Limma
2160	—

(Verified: every Semitone = 16:15, every Limma = 256:243, every Diesis = 25:24; 1080 : 2160 = the octave 2:1.)

CHAPTER VII

Whether, and in what order, the voices of the Muses and the sounds of the Strings ought to be accommodated to the Celestial Spheres

(An et Quo ordine Musarum voces, & Chordarum sonos Caelestibus sphaeris accommodare oporteat)

[Margin: Sirens and Muses, presidents of the celestial orbs.]

[I.] I shall begin from **Macrobius**, who (bk. 2 on the Dream of Scipio, ch. 3) says: *"Plato in his Republic (namely bk. 10), when he treated of the revolution of the celestial spheres, says that single **Sirens** sit upon the single orbs"* — signifying that by the motion of the spheres a song is presented to the divine powers; for *Siren*, in the Greek understanding, means *"one singing to God"* (*Deo canens*). The Theologians too [posit] nine Muses, etc. And a little below: *"That the Muses are the song of the World even the rustics know"* (or, as others read, *"the Etruscans know"*) — who call them **Camoenae**, as if **Canenae**, from *singing* (*canere*); for these the meters were wont to be made with instruments.

[Margin: Strophe and Antistrophe, a symbol of the two heavenly motions.]

In the very **hymns of the gods**, too, by **strophe and antistrophe** the meters were applied to the tuneful verses — so that by the **strophe** the *direct* motion of the starry orb, and by the **antistrophe** the *diverse retrograde* [motion] of the wandering [planets], might be set forth; of which two motions, the first (by nature to be dedicated to God) took its beginning. Concerning the use of strophe and antistrophe and the motion in a circle, for representing the two motions of the heaven, this is held also by **Didymus** and **Victorinus** (in the scholia on Pindar), the ordinary gloss on Pindar, and **Triclinius** (on our Sophocles) — whose words **Mazzoni** reports (*Defense of Dante*, bk. 2, ch. 34); and he reconciles Didymus and Victorinus (who say the strophe was from *right to left*, to show motion from East to West) with the Gloss and Triclinius (who say it was *left to right*) — because the former spoke of the right and left of the *World*, the latter of the right and left of *man*.

[Margin: The number and names of the Muses.]

But returning to the **Muses**, there is no agreement among writers as to their number, names, order, or offices. **Phurnutus** related that there were once **two**, on account of Theory and Practice; **three** [say] Ephorus (in Arnobius), Varro, St. Augustine, and Tzetzes the Grammarian (on Hesiod) — namely Cephiso, Apollonis, and Borysthenis, daughters of Apollo — which **Glareanus** (*Dodecachordon* 2, ch. 14) says was a fiction on account of the **threefold sound**, made by voice, by breath, or by percussion. Yet **four** are posited by **Aratus** (bk. 5), daughters of Jupiter and the nymph Plusia — Arche, Melete, Thelxinoë, and Aoede; and likewise **Cicero** (*On the Nature of the Gods* 3) names four — Thelxiope, Mneme, Aoede, Melete. But **Epicharmus**, at the wedding of Hebe, reckons **seven**, daughters of Pierus and the nymph Pimpleïs — Nilus, Triton, Asopus, Heptapolis, Acheloïs, Tipoplus, and Rhodia. Yet the more common opinion, with **Homer, Hesiod, and Orpheus**, holds there were **nine** Muses, daughters of Jupiter and Mnemosyne; nor did Phurnutus dissent, saying that the **ternary multiplied into itself** (3 × 3) begets the **novenary** number of the Muses. **Hesiod**, in the *Theogony*, reckons their names in this order:

Clio, and Euterpe, and Thalia, and Melpomene,

and Terpsichore, and Erato, and Polyhymnia, and Urania,

and Calliope — who is she that excels all in order.

[Margin: The inventions and offices of the Muses.]

Their inventions and gifts, too, not all reckon alike. Our fellow-countryman **Lilius Gregorius Gyraldus** (*Syntagma* 7, *On the gods of the nations*, under *Musa*) reports these: **Clio** invented history; **Thalia** the art of *planting*; **Euterpe** the flutes; **Melpomene** the *Ode* (song); **Terpsichore** the *Chorea* (dance); **Erato** nuptial

[songs] and dancing; **Polyhymnia** agriculture; **Urania** astrology; **Calliope** poetry. Yet below he interprets them **allegorically**: Urania = the sublimity of intelligence; Polyhymnia = the capacity of memory; Euterpe = the delight of the will; Erato = the love of like things; Melpomene = the profundity of thought; Terpsichore = the exercise of the arts; Calliope = the beauty of eloquence; Clio = the good fame and glory begotten of the foregoing; and Thalia = the germination of the virtues. The same Gyraldus sets forth this Greek epigram, rendered by him into Latin:

> *Calliope showed the hero the art of song;*
>
> *Clio brought forth the sweet-sounding measures of the lyre;*
>
> *Euterpe the resounding song of the tragic chorus;*
>
> *Melpomene moved the barbitons with sweet concord;*
>
> *and pleasing Terpsichore made ready to blow the reeds;*
>
> *but Erato found the gods' delightful hymns;*
>
> *Polyhymnia joined harmony to numbers, and the dances;*
>
> *Urania [sang] the chorus of the stars and the heaven's rotations;*
>
> *the comic life is thine, Thalia, and the manners found out.*
>
> *[Margin: Apollo Musagetes.]*

Apollo himself, moreover, is called **Μουσηγέτης** (*Musagetes*), that is, the **Leader and Choirmaster of the Muses** — as Phurnutus, Macrobius (bk. 2 on the Dream of Scipio, ch. 3), and **Proclus** (on Plato) relate; whose words are: *"Apollo is worshipped as Musagetes, because he is the **unity** [tending] to harmony in the universe; and he is the choirmaster of the Muses of the whole novenary number — by which two [the unity and the nine] the whole world is bound together with indissoluble bonds."*

[II.] As for what pertains to the **order of the Muses with the celestial** [spheres]…

> *[The catchword **"caele-"** points to p. 522 (PDF 557), which continues ¶II on the order of the Muses and the spheres, within Chapter VII.]*

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*(printed p. 522 — **Chapter VII** continued. The order of the Muses with the spheres is expounded from Cicero's Dream of Scipio and Macrobius, with Calliope as the ninth, the symphony of all the spheres; three differing assignments (Glareanus, Ficino and Zarlino, Gyraldus) are set out in a synopsis table. Riccioli then gives his verdict that these analogies were fixed "more elegantly than truly," supported by four reasons against the scheme, the fourth beginning at the page's end.)*

CHAPTER VII

(continued — the order of the Muses with the celestial spheres)

[Margin: The order of the Muses in the heavens. — Cicero's opinion. — And Macrobius's.]

[II.] As for what pertains to the **order of the Muses** compared with the celestial [spheres], various [authors] think variously. **Eudoxus** indeed determined nothing of this; nor did **Cicero** in the *Dream of Scipio* — though he indicated his mind obscurely in these words: *"Nine orbs, or rather **globes**, are all connected: of which one — the outermost and celestial, embracing all the rest — is the supreme God himself, restraining and containing the others, in which are fixed those everlasting courses of the turning stars. Beneath it are the **seven**, turned backward by a motion contrary to the heaven's,"* etc.; and a little below: *"And in the lowest orb the Moon is kindled by the Sun's rays"* (the lowest of the moving bodies); for soon after he adds: *"For that*

*which is the middle and ninth, the **Earth**, neither moves and is the lowest, and toward it all things are borne by their own weight."* Wherefore, by Cicero's mind, the **lowest** Muse is to be assigned to the **Earth**, the **highest** to the sphere of the Fixed stars (or to the whole heaven, as containing in itself all the others' motions).

This passage of Cicero **Macrobius** illustrates (bk. 2 on the Dream of Scipio, ch. 3), teaching that the Muses are to be ascribed to the celestial spheres in exactly the order Hesiod enumerated — so that **Clio** [is given] to the lowest, **Urania** to the eighth sphere, and **Calliope** is the most beautiful voice, as rising from the concord of all the spheres. Macrobius's words: *"The Theologians too held there were nine Muses — the eight musical songs of the spheres, and one greatest [harmony] containing [them], which consists of all; whence Hesiod in the Theogony calls his eighth Muse **Urania**, because after the seven wandering [spheres] beneath, the eighth, the starry sphere placed above, is by its own name called heaven. And to show there is a **ninth** and greatest [Muse], which the concordant whole of sounds produces, he added: 'Calliope, and she is the most excellent of them all' — showing by the name that the very sweetness of voice is called the ninth Muse (for Καλλιόπη is the Greek for 'best voice'); and to mark more closely that she is the one consisting of all, he assigned to her the word of the whole, namely 'most excellent of all.'"*

[Margin: Glareanus's opinion.]

The same of Calliope held **Glareanus** (*Dodecachordon* 2, ch. 12): *"that the ninth orb must be understood as the symphony conflated from these eight sounds, and on this account called Calliope, as Plato philosophizes of the Muses in the Republic"* — not rashly, in the judgment of all the learned; the same is Cicero's treatment in the Dream of Scipio. And (ch. 14) he affirms, from Plato (*Republic* 10), that the **Sirens** assigned to the single spheres are called the **eight Muses** on account of the eight sounds, and the Symphony conflated from these is called by him the **ninth Muse**, and by Hesiod **Calliope**, as the most excellent of all; and he praises **Cristoforo Landino** (above Servius) for interpreting, of this universality and excellence of voices comprehended in Calliope, that verse of Virgil (*Aeneid* 9): *"You, O Calliope, I pray, breathe upon me as I sing"* — where the best of Poets, using the **plural**, is addressing Calliope as her who comprehends all the others in or under herself.

In the **System of the World**, then, [Glareanus], together with Hesiod, assigns to the **Moon** Clio; to **Mercury** Euterpe; to **Venus** Thalia; to the **Sun** Melpomene; to **Mars** Terpsichore; to **Jupiter** Erato; to **Saturn** Polyhymnia; to the **Fixed stars** Urania; and to the last orb of all, **Calliope**.

[Margin: Marsilio Ficino's opinion.]

But **Marsilio Ficino** (in [his commentary on] Plato's dialogue *On Poetic Fury*), and subscribing to him **Zarlino** (*Harmonic Institutions* 2, ch. 19), apply: to the **Moon** Thalia; to **Mercury** Euterpe; to **Venus** Erato; to the **Sun** Melpomene; to **Mars** Clio; to **Jupiter** Terpsichore; to **Saturn** Polyhymnia; to the **Fixed-stars** sphere Urania; and over the single spheres and all together they set **Calliope**, to signify the concord arising from them all.

But **Gyraldus** (*Syntagma* 7, *On the gods of the nations*, under *Musa*) attributes **Thalia** to the **Earth**, as silent and having no concord; to the **Moon** Clio and the **hypodorian** mode; to **Mercury** Calliope and the **hypophrygian**; to **Venus** Terpsichore and the **hypolydian**; to the **Sun** Melpomene and the **dorian**; to **Mars** Erato and the **phrygian**; to **Jupiter** Euterpe and the **lydian**; to **Saturn** Polyhymnia and the **myxolydian**; to the **Fixed stars** Urania and the **hypermyxolydian**. To which opinion subscribes, in Mersenne (on Genesis 4, verse 21, p. 1704), an **unnamed author** in these verses:

Persephone [a — the Moon] and Clio breathe; therefore the hypodorian is born:

whence the Proslambanomenos [b — the lowest string] generates its origin.

And next [Calliope] gives the hypophrygian chord that follows,

which Calliope herself brings forth — the interpreter [c — Mercury] himself brings it forth — of the gods.

The third shows the hypolydian beginnings on the strings:

Terpsichore meets it; kindly Paphis [d — Venus] sets it in order.

Melpomene and Titan [e — the Sun], believe me, appoint the mode

which is said to stand in the fourth place, the Dorian.

Erato would prescribe the fifth string to the Phrygian —

Mars too, ever loving battles, not peace.

The Lydian thou shalt have of Euterpe and of Jupiter;

she, sweetly holding [it], bade that to be the sixth string.

Saturn drives Polyhymnia's seventh string,

the principle whence the myxolydian takes its rise.

And while Urania searches out the eighth, her friend,

the hypermyxolydian turns the pole by art.

*[The margin keys the planets: **a** = Moon, **b** = Proslambanomenos (the lowest string), **c** = Mercury, **d** = Venus, **e** = Sun. The verses encode exactly Gyraldus's assignment of Muse, planet, and mode. A few phrases are awkward in the original Latin mnemonic; "Prosmeledes" stands for the Proslambanomenos, "Paphis" = Venus, "Titan" = the Sun.]*

It pleases me, therefore, to bring these opinions together into one synopsis, in the following table:

The Distribution and Place of the Muses in the System of the World

(Musarum Distributio et Locus in Systemate Mundi)

Sphere (Glareanus)	Glareanus	Ficino & Zarlino	Gyraldus & others (in Mersenne)
The Ninth, or the eight together	Calliope	Calliope	—
Fixed stars	Urania	Urania	Urania
Saturn	Polyhymnia	Polyhymnia	Polyhymnia
Jupiter	Erato	Terpsichore	Euterpe
Mars	Terpsichore	Clio	Erato
Sun	Melpomene	Melpomene	Melpomene
Venus	Thalia	Erato	Terpsichore
Mercury	Euterpe	Euterpe	Calliope
Moon	Clio	Thalia	Clio
Earth	—	—	Thalia

(The "—" marks where the original sets a cross, i.e. no Muse assigned to that place.)

And so all agree in the distribution of three Muses — **Urania, Polyhymnia, and Melpomene** [= the Fixed stars, Saturn, and the Sun]; in the rest they do not all agree.

[Margin: Our opinion, and a censure from Aristotle and Pliny.]

[III.] You will now ask our opinion: to which indeed we reply that these analogies and harmonies have been determined by the aforesaid authors **more elegantly than truly** — to use Aristotle's words (*On the Heaven* 2, text 52), or to say with **Pliny** (bk. 2, ch. 22): *"Saturn is moved by the Dorian, Mercury by [its] phthongus,*

Jupiter by the Phrygian," and the like in the rest, with a subtlety **more pleasant than necessary**; and Macrobius himself (bk. 2 on the Dream of Scipio, ch. 4) says that to pursue these things more subtly *"belongs to one showing off, not to one teaching."* And of the same opinion are **Glareanus** (*Dodecachordon* 2, ch. 13, at the end), **Zarlino** (*Harmonic Institutions* 2, ch. 29, at the end), **Mersenne** (on Genesis 4, p. 1704), and **Kircher** (*Musurgia* 10, p. 381).

[Margin: Our reasons.]

But the **reason that moves me is not a single one**.

First — how **inconstant** the tradition has been concerning the **number, names, and properties** of the Muses, we have made plain enough at number 1 [in ¶I].

Second — the **order of the celestial spheres** itself was various among the older Astronomers, as is clear from the variety of world-systems already expounded by us (bk. 3, ch. 6, and bk. 9, sect. 3); wherefore you would not even satisfy **Macrobius** by assigning **Thalia and Euterpe to Venus and Mercury**, since he allows those planets to be carried **above the Sun** as well.

Third — if the **Sun** is the leader of the Muses (and so called **Musagetes**), how does one of the Muses preside over the Sun's own sphere — or [is she there] as his vicar or helper? Again, if **no ninth sphere** is granted, distinct from the others as a Prime Mobile, but **each planet is moved by a single spiral motion**, there will be **no sphere** to which Calliope (or any other ninth Muse) can be assigned; and much less is a Muse to be given to the **Earth**, since it does not move.

Fourth — how, with the **pla**[nets]…

[The text breaks off mid-word at "Quartò quomodo cum Pla-"; the fourth reason continues on p. 523 (PDF 558), within Chapter VII.]

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*(printed p. 523 — **Chapter VII** continued. The case against the Muse-sphere scheme concludes — better to recognize the Peripatetic Intelligences as governesses of the heavens — and a sub-section opens on the analogy of the strings and the celestial spheres. The celestial Lyre's disputed invention and string-number are surveyed; then the positive case for real celestial harmony from Cicero and Macrobius, the cause of the authors' disagreement (the twofold motion and the orbs' size), and the First opinion, which gives the higher pitch to the swifter superior spheres.)*

CHAPTER VII

(continued — the fourth reason; then the Analogy of Strings and Spheres)

…**[Fourthly]** — how, with the **Platonic** opinion, will the Muses and the Sirens stand together, if Greece has *elsewhere* fabled that the **Sirens were conquered by the Muses** in a contest of song? It is better, therefore, these fables being dismissed, to recognize in the place of the Muses the **Peripatetic Intelligences** — which the Theologians too acknowledge — as the **governesses of the heavens**.

On the Analogy of the Strings and the Celestial Spheres

(De Chordarum & Caelestium sphaerarum Analogia)

[Margin: The inventor of the Lyre — Mercury or Apollo?]

[IV.] Before we treat of the strings of the **celestial Lyre**, it must briefly be inquired who was the inventor of the Lyre, and of how many strings. For if we believe **Diodorus Siculus**, **Mercury** devised the three-stringed

Lyre, but **Orpheus** made it four-stringed; while **Terpander** (by Plutarch's testimony, in the opuscule *On Music*) added to it the laws and the names of the strings — though Suidas, Boethius (bk. 1, ch. 20), and Aristotle (sect. 19, problem 32) attribute the seven-stringed Lyre to him. But **Macrobius** (*Saturnalia* 1, ch. 19) relates that the **tetrachord** [four-stringed] was attributed to Mercury, to signify as many regions of the world or seasons of the year; and he adds that *"by the seven-stringed Lyre of Apollo the motions of as many celestial spheres are given to be understood, over which Nature set the Sun as governor."* But in his poetic *Astronomicon*, **Hyginus** affirms that Mercury was the inventor of the seven-stringed Lyre, on account of the number of the seven **Pleiades** (or Atlantides); and that, when Mercury had driven off Apollo's cattle, to satisfy Apollo (who pursued him) he complied and conceded that **Apollo** should name himself the inventor of the seven-stringed Lyre — which was moreover increased by Apollo with that **wand** by which, a little after, Mercury (reconciling two serpents) was wont to carry the **caduceus** in place of the lyre.

But even before Hyginus, **Homer**, in the hymn he dedicated to Mercury, acknowledges him the inventor of seven symphonies through as many strings made from birds' entrails, in this verse:

> *Ἑπτὰ δὲ συμφώνους οἴων ἐτανύσατο χορδάς — that is,*
>
> *"And he stretched seven consonant strings from the entrails of sheep."*

With whom agrees **Horace** (*Odes* 3, Ode 11), addressing Mercury and his lyre (*testudo*):

> *"And thou, O lyre, skilled to resound with seven strings."*

In imitation of this — or to imitate the seven symphonies, whether celestial or human — **Pan** is believed to have first compacted seven reeds into one wind-instrument; such a pipe **Ovid** mentions (*Metamorphoses* 2) in this hemistich: *"...the pipe unequal with seven reeds";* and **Corydon** in Virgil (*Eclogue* 2): *"I have a pipe compacted of seven unequal hemlock-stalks."* Though that **Menalcas** in **Theocritus** boasts of a pipe of nine tones (or reeds) made by himself, in this verse:

> *Σύριγγ' ἐποίησα καλὰν ἐγὼ ἐννεάφωνον — that is,*
>
> *"I myself made a beautiful syrinx of nine tones."*
>
> *[Margin: The 7 Species of the Diapason.]*

But returning to the **seven-stringed Lyre**: **Glareanus** (*Dodecachordon* 2, ch. 12) judges that by its seven strings are signified the **seven species of consonances** into which the Diapason is divided, and which reign in the two octochord instruments; and to this he draws that verse of Virgil (*Aeneid* 6):

> *"And likewise the Thracian priest [Orpheus] in the long robe accompanies [them] with his numbers, the seven distinctions of tones."*

And those species of the Diapason are: Diapason-diapente; Disdiapason; Diapason-with-ditone; Diapason-with-semiditone; Diapason-with-greater-hexachord; and Diapason-with-lesser-hexachord — unless for these you understand the **seven simple consonances** which we enumerated in ch. 4, table 1.

> *[Margin: Cicero's opinion on the high sound of the supreme sphere.]*

[V.] But whatever may be the case about the inventor and purpose of the seven- (or even nine-) stringed Lyre or pipe, it is certain that **not a few authors, and not to be despised, recognized in the celestial spheres the intervals and distinctions of grave and acute voices**, and so adapted to them the strings of the Lyre or of the harmonic system. In which, from the Platonic source, **Cicero** led the way in the Dream of Scipio: when Scipio asks, *"What is this, I say — what so great and so sweet a sound that fills my ears?"*, he brings in his father **Paulus** answering in these words: *"This is that [sound] which, joined by intervals unequal yet distinguished in due proportion, is produced by the impulse and motion of the orbs themselves; which, tempering high with low, makes equable concords. For the motions cannot be set going in silence; and nature brings it about that the extremes sound, on the one side gravely, on the other acutely. For which cause*

*that **highest course of the starry heaven, whose revolution is swifter, is moved with a high, excited sound; but the lowest, the Moon's, with the gravest.** For the Earth, ninth, remaining immobile, always clings in the lowest seat, embracing the middle place of the world. But those eight courses — in which the same force is [shared by] two — produce **seven sounds distinct by intervals**, which number is the **knot of nearly all things**; which learned men, imitating with strings and songs, have opened for themselves a return to this place."*

[Margin: Macrobius's opinion.]

Explaining these words, **Macrobius** (bk. 2, ch. 4) teaches that from a **great and swift collision of the air** an acuter [higher] sound is made, but from a slow and more sluggish one a graver [lower] — which he confirms by the example of a rod [swung] through the air, and of strings (which, stretched by a tighter pull, sound higher; by a looser, lower), and of flutes and pipes (whose narrower holes, or those more remote from the blower's mouth, give a graver sound than the wider or nearer). Whence he concludes that to the **supreme starry orb** — both for its magnitude, by which it extends immensely, and for the greatest velocity of its diurnal motion to the West — the **acutest** [highest] sound is due; but to the **Moon**, for the narrowness of its orb and the slowness of its motion, the **gravest** [lowest]; and, of the other spheres described by Tully, that the **Earth** is immobile, the **other eight** mobile — yet that there are only **seven** sounds, because **Mercury and Venus move with the same motion as the Sun**, and therefore Cicero said the **force of the two is the same**.

[Margin: Glareanus's exposition of Cicero and the text.]

But **Glareanus** (*Dodecachordon* 2, ch. 12) thinks the passage of Cicero is **corrupt** and not sufficiently understood by Macrobius, and that the word **"seven"** should be referred not to the *sounds* but to the **intervals** — so that the eight courses produce [eight] sounds, but distant from one another by **seven intervals**; or rather it should be read: *"But those courses, in which the force of the extremes is the same, produce sounds distinguished by seven intervals."* For from Music it is read that **eight phthongi** (or principal voices) produce **seven intervals**, that is, the seven species of the Diapason.

[Margin: The cause of the diversity of opinions in this matter.]

[VI.] Now from the sources of Plato and Cicero **three different opinions** have flowed: of which the **First** assigns to the **supreme sphere** the **highest** [acutest] of the strings — that is, either the **Mese** (the highest of the prior nine strings of the harmonic system) or the **Nete hyperbolaeon** (the highest in the ancient 15-string system). The **Second** assigns to the supreme sphere the **gravest** [lowest] string — namely the **Proslambanomenos**, or at least the **Hypate hypaton**. The **Third**, finally, either adheres to both under differing considerations, or strives to reconcile them, though leaning more to one than the other.

The cause of the diversity of these opinions is, on the one hand, the **twofold motion apparent in the heaven**, and on the other, the **diverse distance and magnitude of the orbs**. For if the **common motion** (called the Prime Mobile's, toward the West) be regarded, the **Fixed stars move faster** than Saturn, and Saturn than the lower planets; and by that reckoning the **highest** sound is due to [the Fixed sphere]. But if the **proper apparent motion** toward the East be regarded, slowest is the course of the Fixed stars, then Saturn, etc.; wherefore, for this [smallness] of motion, the **gravest** sound seems assignable to them. And by **distance** — since the length of a string and the magnitude of bodies give a graver sound than shortness and smallness (as Aristotle teaches, sect. 19, problem 8) — it seems that to the spheres and orbs **more remote from the Earth** a graver [lower] sound should be attributed.

It is moreover **disputed** whether the string giving the gravest sound ought to be placed in the **innermost** place of the musical System and of the tetrachords (the highest in the supreme place), or rather the **gravest in the supreme** and the **highest in the lowest** — on which Glareanus disputes acutely (*Dodecachordon* 1, ch. 5, and bk. 2, ch. 8).

[Margin: The 1st Opinion — of Cicero, Macrobius, and Guido of Arezzo.]

[VII.] The **First opinion** is that of **Guido of Arezzo**, whom many have followed; who (as is clear from the systems set forth in ch. 5, after number 3) placed in the **innermost** place the **gravest** string [Gamma], to which succeeds the Proslambanomenos, and in the **supreme** degrees of the Scale the **higher** sounds — which agrees with **Cicero** (*Republic* 6, the Dream of Scipio) and **Macrobius**: namely, that the **superior bodies**, as swifter in the diurnal motion, give a **higher but lesser** sound, but the **inferior**, as slower, a **greater but graver** [lower] sound…

[The text breaks off at "graviorem so-" (sonum); ¶VII continues on p. 524 (PDF 559), within Chapter VII.]

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*(printed p. 524 — **Chapter VII** continued. The First opinion (highest pitch to the supreme sphere) is completed with its followers and supporting analogies from the cithara, organ, and human voice, plus Mersenne's cabalistic ten-stringed Cithara of David set out in a table. Then the Second opinion is given: the superior spheres, being slower and bigger, sound the graver note — held by Nicomachus, Servius, Poliziano, and Boethius, and judged "much more probable" by Glareanus, who offers two diagrams.)*

CHAPTER VII

(continued — the First opinion concluded, the Cithara of David, and the Second opinion)

…[the superior bodies, as swifter, give a higher but lesser sound, the inferior a greater but graver] — **which opinion was followed** by **Guglielmo Philander** (on Vitruvius bk. 5, ch. 4, attributing it to the ancients), **Gyraldus** (*Syntagma* 7, under *Musa*), **Valla** (on Pliny bk. 2, ch. 22), **Fr. Athanasius Kircher** (*Musurgia* 10, p. 393), and the **unknown Author** in Mersenne (on Genesis 4, verse 21, p. 1704). For they give the **gravest** string to the **Moon or Earth**, and the **acutest** to **Saturn or the sphere of the Fixed stars**.

To this favors, first, the **order of the strings in the Cithara**, which is *contrary* to the order in the **Lyra**: for in the Lyra the gravest and thickest string is in the **supreme** place (whether three-, four-, or six-stringed), but in the Cithara the gravest and thickest is in the **lowest** seat, and the acutest in the supreme — the order our **organs** still keep, in which the thicker pipes are set in the lower, the thinner in the upper row. And the **human voice**, too, more often begins not from the highest but from the lowest [note] and a graver sound, and gradually by a natural transition ascends to the higher; for the **graver voice is, as it were, the foundation**, without which the higher and acuter voices seem to degenerate into chattering, unless sustained by the strength of a graver voice. For though the highest voice may seem to soothe the hearing more sweetly, yet (as Glareanus noted above) no song is more pleasant than when the **lowest, firm voice** sounds; and the higher voices accomplish almost nothing without the **Bass** of the lower (so Julius Pollux names the graver voices); nor today is the **Diatessaron** admitted unless propped from below by a **Diapente** (the fifth), or even a third.

Mersenne too — although (on Genesis 4, p. 1704) he repudiates such fictions — yet (p. 1705), from the **arcana of the Cabalists**, describes the **ten-stringed Cithara of David** by the following analogy, in which he assigns the **acuter** [higher] sound to the supreme spheres and spirits. Behold it in the table:

The strings of the Davidic Cithara, according to the Cabalists

(Davidicae citharae nervi iuxta Cabalistas) — ascending from the lowest string (1) to the highest (10)

String	Greek name	Note	Body	Spirit
1	Parhypate meson	F (fa ut)	the Elements	Blessed soul (*Anima beata*)
2	Lichanos meson	G (sol re ut)	Moon	Angels

String	Greek name	Note	Body	Spirit
3	Mese	A (la mi re)	Mercury	Archangels
4	Paramese	B (fa / b mi)	Venus	Principalities
5	Trite diezeugmenon	C (sol fa ut)	Sun	Virtues
6	Paranete diezeugmenon	D (la sol re)	Mars	Powers
7	Nete diezeugmenon	E (la mi)	Jupiter	Dominions
8	Trite hyperbolaeon	F (fa ut)	Saturn	Thrones
9	Paranete hyperbolaeon	G (sol re ut)	Fixed stars	Cherubim
10	Nete hyperbolaeon	A (la mi re)	Prime Mobile	Seraphim

(The "Body" column stands in the original as planet-symbols, here given by name; it runs in the ascending Ptolemaic order — Elements, then Moon, Mercury, Venus, Sun, Mars, Jupiter, Saturn, the Fixed stars, and the Prime Mobile. So the higher strings answer to the higher spheres and higher angelic orders — the First opinion's arrangement.)

[Margin: The 2nd Opinion.]

[VIII.] The **Second opinion** assigns to the **superior spheres** the **graver** [lower] sound — either because it thinks them slower (regarding their proper motion), or because, even if the greater velocity of the common [diurnal] motion lends them acuteness, yet the **length of the interval and the magnitude of the bodies** make a graver and greater sound — just as in the strings of many-stringed instruments, where the thicker and greater string begets a **graver but more sonorous** voice.

[Margin: Nicomachus's, and Servius the Grammarian's.]

Of this opinion was **Nicomachus** (in the *Enchiridion of Harmonics*, as Boethius and Philander report), who accordingly ascribed the **Hypate to Saturn** and the **Nete to the Moon**; and **Servius Honoratus**, who explains Virgil's *"through the friendly silences of the silent Moon"* thus: *"'Silent,' he says, [means] the Moon — either it signifies night, in the poetic manner; or he spoke by a physical reason. For there are seven circles — of Saturn, Jupiter, Mars, the Sun, Venus, Mercury, the Moon; and the first, that is **Saturn, sounds vehemently**, the rest less according to their order, as we have heard in the Cithara, whose last string sounds less."* — where he calls the *last* string in the cithara that which is **nearer to our ears**, and in that respect compares it with the Moon; not according to the order of high and low position (for the Moon is *down*, Saturn *up*), but contrariwise: in the cithara the acutest, thinnest, least-sounding string is *up*, the gravest *down*.

Wherefore **Angelo Poliziano** (*Miscellanea*, last chapter) neither understood Servius nor justly reproved him, when he says: *"This will hold if you take the 'last' not as the lowest but as the highest — which in Greek is called Hypate — whence a graver sound is roused; just as from the other extreme, the thinnest string (which I would rather call the last), an acute voice is uttered, because it is stretched fully."* But Poliziano is more justly reproved by **Glareanus** (*Dodecachordon* 2, ch. 8), who teaches that Servius spoke **truly and according to nature and the series of the celestial orbs** (if there were sound there), and indeed according to the **Cithara's strings** — for in it the **Hypate**, the gravest, holds the **lowest** place, the acutest the highest, which, even if stretched the most, is not for that reason heard the most. *"For granted,"* says Glareanus, *"that in one and the same string the acuteness strikes the hearing more when it is stretched tight than the gravity when stretched looser; yet among different strings, the greater and thicker is heard more and makes a greater sound than the thinner, even if the latter be stretched more and sound higher — just as the voice of a well-voiced man is heard more than the wailing or clamor of a little boy. Hence too the higher voices accomplish almost nothing without the **Bass** and firmness of the graver."*

[Margin: Poliziano's [agreement]; and Boethius's.]

Yet it appears that **Poliziano** felt rightly *in this*, in conceding the graver sound to the **Hypate**, and by placing it in the **highest place** subscribed to Servius, who assigns this to **Saturn**. Of the same opinion, finally, was **Severinus Boethius**: for he not only acknowledged the Hypate as the gravest of the strings, but ascribed it to the **superior spheres**; for (bk. 1 of his *Music*, ch. 20) he says of the strings: *"And among these, the one that was gravest was called **Hypate**, as if greater and more venerable — whence they call **Jupiter** also **Hypatos**, and name the **Consul** by the same name for the excellence of his dignity; and it [the Hypate] is attributed to **Saturn**, for the slowness of his motion and the gravity of his sound."* So (bk. 4, ch. 10) he places the **Proslambanomenos in the supreme place** and the **Nete in the lowest**.

[Margin: Glareanus's opinion.]

But even **Glareanus**, Boethius's follower — though he **absolutely denies that there is any sound in the heavens**, or any true principle of Symphony (*Dodecachordon* 2, ch. 14, from the paragraph *"Moreover"* to the end) — nevertheless, **on the supposition** that there were some sound in heaven, or that one should philosophize of the celestial bodies as of terrestrial sounding ones, leans to **Boethius's view**. For (*Dodecachordon* 1, ch. 5) he says: *"I am not unaware that to many this order of the strings posited by Guido seems **inverted** and altogether against the natural course of the heaven (whence this scheme was taken) — inasmuch as the **superior bodies of the celestial orbs give a graver sound, being the greater**; and this the divine Boethius too seems everywhere to have observed in his demonstrations."* And a little after: *"Whether, then, [we take them] inverted, so that the superior be the graver — namely that we place the **hypaton tetrachord in the supreme place, with the Proslambanomenos** — which Severinus several times did, either for the name **Hypatos** (which sounds 'principal'), or because the first teachers of this art so judged. For many old instruments — three-, four-, and six-stringed — still have it so, etc."* And (bk. 2, ch. 8) he says that Servius spoke truly and according to nature and the series of the celestial orbs (if there be sound there); and it is clear from what was said that Servius gave **Saturn the gravest and most vehement sound**. And (ch. 13), having recalled Cicero's opinion, withdrawing from it he adds: ***"Others, on the contrary, ascribe the acuter sounds to the inferior bodies, and the graver (or greater) to the superior — because greater bodies give a greater sound, smaller a smaller; which opinion seems to me much more probable**, since the heaven, like this sensible world, has bodies [proportioned so]."* And he subjoins a **twofold diagram**: one **Ciceronian**, in which he assigns the **shorter and acuter strings to the superior spheres**; the other **Boethian**, in which …

[The text breaks off at "alterum Boëtianum, in quo" (the other, Boethian [diagram], in which…); the two diagrams and the rest follow on p. 525 (PDF 560), within Chapter VII.]

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*(printed p. 525 — **Chapter VII concludes, then Chapter VIII opens.** The Second opinion is finished and the Third (Zarlino's indifferent position, giving both a Ciceronian and a Boethian system) is stated, with a wide comparison table of the celestial strings assigned by each authority. Riccioli's own opinion follows: the analogies are pursued "more wittily than truly," there being no real sound in the heavens, supported by five reasons against fixing strings to spheres. Chapter VIII then opens, on fitting the harmonic intervals to the distances of the stars, beginning with the ancient authorities.)*

CHAPTER VII

(conclusion — Bettini, Zarlino's third opinion, the comparison table, and the Author's verdict)

[Margin: And Bettini's.]

…[the other, **Boethian**, diagram], in which he assigns the **longer and graver strings to the superior** [spheres] — which diagrams we shall set out below in a table with the rest; but in ch. 14 he chose **only the**

Boethian. Likewise **Mario Bettini** (*Apiarium* 10, Progymnasma 1, Proposition 1), inasmuch as he applies the **Hypate to the Fixed stars, the Parhypate to Saturn, and the Nete to the Moon**. And the **order of the Lyra** favors this opinion, in which the thicker, graver-sounding string holds the **supreme** place, and the acuter-sounding string the **lowest** — which is also kept in most instruments.

[Margin: The 3rd Opinion.]

[IX.] The **Third opinion** shows itself **indifferent** — in which is **Zarlino** (*Harmonic Institutions*, part 2, ch. 29), who posits **two systems**: one according to **Cicero** (but assigning the **Mese to the Fixed stars**, and the **Proslambanomenos to the Moon**); the other according to **Boethius**. Yet in the aforesaid three opinions there remains some variety as to the **whole order of the strings**, as will appear more clearly from the following tables.

The Order of the Celestial Strings

(Chordarum Caelestium Ordo) — each celestial sphere (read downward, Fixed stars to Earth) set against the string assigned to it by each authority

A. According to Cicero & Macrobius *(Ad mentem Ciceronis & Macrobij)* — the higher the sphere, the higher the string

Sphere	Glareanus, Zarlino & Gyraldus	Philander	Kircher
Fixed stars	Mese	—	—
Saturn	Lichanos Meson	Nete	Nete
Jupiter	Parhypate Meson	Paranete	Paranete
Mars	Hypate Meson	Paramese	Paramese
Sun	Lichanos Hypaton	Mese	Mese
Venus	Parhypate Hypaton	Lichanos Hypaton	Lichanos Hypaton
Mercury	Hypate Hypaton	Parhypate Hypaton	Parhypate Hypaton
Moon	Proslambanomenos	Hypate Hypaton	Hypate Hypaton
Earth (*Tellus*)	—	—	Proslambanomenos

B. According to Nicomachus & Boethius *(Ad mentem Nicomachi & Boëtij)* — the higher the sphere, the **lower** (graver) the string

Sphere	Glareanus & Zarlino (string / gamut-letter)	Bettini (string / syllable)
Fixed stars	Lichanos Hypaton — A re	Hypate — Ut
Saturn	Hypate Meson — B mi	Parhypate — Re
Jupiter	Parhypate Meson — C fa ut	Lichanos — Mi
Mars	Lichanos Meson — D sol re	Mese — Fa
Sun	Mese — E la mi	Paramese — Sol
Venus	Trite synemmenon — F fa ut	Trite — Re (re)
Mercury	Paranete synemmenon — G sol re ut	Paranete — Mi (mi)
Moon	Nete synemmenon — a la mi re	Nete — Fa (fa)

Sphere	Glareanus & Zarlino (string / gamut-letter)	Bettini (string / syllable)
Earth (*Tellus*)	—	—

(Transcribed as printed. In the Boethian half the original does not re-print the planet-symbols beside each row, but the alignment is fixed by the text just above — Bettini "applies the Hypate to the Fixed stars, the Parhypate to Saturn, the Nete to the Moon" — and the Glareanus–Zarlino column runs in exact parallel, its gamut ascending A-a as the sphere descends Fixed-Moon. The Greek string-names in Glareanus's Boethian diagram are his own labeling and do not match the usual letter-assignments. Below the table the original adds: "Add to these the [Cithara of David] which I delivered from the Cabalists at the end of number 7" — i.e. the table on p. 524.)

[Margin: The Author's opinion. — Macrobius's excellent judgment. — Glareanus's definitive judgment. — Our 1st reason… 2nd reason.]

[X.] Now **what our own opinion is** in this matter can already be clear from what was said from the beginning of number 3 — namely, that these things are inquired into **more wittily than truly**, with a subtlety **more pleasant than necessary**, as we said there with **Aristotle** and **Pliny**. And **Macrobius** (bk. 2 on the Dream of Scipio, ch. 4), having this controversy in hand, prudently concluded thus: *"To illuminate, as I think, the obscurity of Cicero's words, this brief treatise on Music will suffice, with what brevity was allowed. For to hunt out the **Netes and Hypates** and the names of the other strings, and the minute subtleties of tones and limmas — and what in the sounds is taken for a letter, what for a syllable, what for a whole name — belongs to one **showing off, not teaching**. For not because Cicero here made mention of music must one therefore run through all the treatises that can be made on Music, which (so far as my opinion bears) I judge to have no end."* In imitation of whom, I believe, **Glareanus** (*Dodecachordon* 2, near the end of ch. 13) concluded: *"Not without reason did this notion seem to Aristotle more pleasant to say than likely to be true";* and a little after: *"But this indulgence is granted to antiquity, which judged these things in some way fit to raise human minds to the contemplation of celestial things."* **Zarlino** too (part 2, end of ch. 29) concludes with Pliny that these are investigated with a charm **more subtle than necessary**; and **Marin Mersenne** (on Genesis 4, verse 21, p. 1704) repudiates them all as **fictions** put forth without sufficient proof. But long before, **St. Ambrose** (in the book *On Isaac and the Soul*, ch. 7) denied that such fancies of a **concord of the heavens** find credence — though, for the pleasantness of the discourse, they are heard with favor and sweetness, as we showed (ch. 2) from his own words.

Nor indeed does that reason move me **alone** — that there is **no real sound in the heavens** (for what I think of this I said in the same ch. 2) — but:

First, that even if there were some sound, yet the **truer order of the Planets** is not the one those authors suppose. And indeed it seems strange that Fathers **Mersenne** and **Kircher**, more recent Astronomers, should place **Mercury nearer the Moon than Venus** — unless you say they spoke from the opinion of the ancients. But since these two planets [Mercury and Venus] are carried now below, now above the Sun, do they not measure out by their own motions their whole portion of that system, which runs from their Apogee to their Perigee? Likewise **Mars**, who sometimes comes nearer the Earth than the Sun, himself by his own ascent and descent occupies a great part of that interval through which the Sun, Venus, and Mercury rise and fall, and so mingles himself with them — so that you cannot assign him a distinct string or interval.

Secondly: Since it is more probable that the **planetary heavens are fluid**, and that the planets move by themselves in them, and not by the motion of [solid] orbs (as said in sect. 1, ch. 2, qu. 2)… a greater sound could indeed be granted to the **sphere of the Fixed stars** (as solid, and as to a greater body); but a greater [sound] **cannot** be granted to Saturn, Jupiter, and Mars than to the Sun, since the **Sun is greater than those three planets**.

[Margin: 3rd reason. — 4th reason. — 5th reason.]

Thirdly: Since there are **eight different species of harmonic Systems** — as I taught from **Martianus Capella** (ch. 5, at the end of the Chromatic and Enharmonic system) — it is **very uncertain which of those three** [genera — Diatonic, Chromatic, Enharmonic] belongs to the celestial system.

Fourthly: If it were free to **play** in these matters, one would have to take account both of the **Satellites of Saturn and Jupiter**, and of the diverse position the planets hold at the **Apogee and Perigee** of the Eccentric and Epicycle, and of the different [reckoning] which the **Fixed stars** hold.

Fifthly: As for the **velocity and slowness of motion** — since [the planets] really move by another real motion than toward the West, and by [their] slowness the lower [bodies] seem to fall back toward the East — the **higher** [planets] ought to be given the **acuter** [higher] sound, as absolutely swifter in this [diurnal] motion (except when they are **Retrograde**); but on the other hand, the **three superior Planets**, by the greater bulk of their body, would give a **graver** [lower] sound than the three inferior. **Therefore one reason destroys the other**; and so this analogy — sung with a certain oratorical and academic charm — **loses its concord in the very discord** — to say nothing of the disagreement of the authors in distributing the strings to the heaven.

CHAPTER VIII

And by what reckoning the intervals of the Harmonic System ought to be fitted to the distances of the Stars

(Et Qua ratione Distantijs Siderum Intervalla Systematis Harmonici debeant accommodari)

Achilles Tatius (in the *Isagoge* to the *Phaenomena* of Aratus, ch. 7, "On the order of the spheres") says [that the ancients treated of the celestial harmony]. Of their **harmonic motion** there treat — in the *Canon*, as has been said — **Aratus**, and **Eratosthenes** in the *Mercury* [*Hermes*]; nor were **Hypsicles, Thrasyllus, and Adrastus of Aphrodisias** the [first] authors of this doctrine, but the **Pythagoreans**, who hold that the universe is moved by [number] and order. **Boulliau** [Bullialdus] too, in the prolegomena to his *Astronomia Philolaica*, [reports] under the name of **François Viète** a *Harmonicon caeleste*, which **Pierre Dupuy** (Puteanus) gave to Fr. **Marin Mersenne** to use, and to him indeed …

*[The opening lines of Chapter VIII are clipped at the column's left edge in the original scan; the gist is secure (a roll-call of ancient and modern authorities on celestial harmony), with a few connective words supplied. The text breaks off at "huic ve-" - catchword **"rò"** (verò); it continues on p. 526 (PDF 561), within Chapter VIII.]*

(printed p. 526 — Chapter VIII (fitting the harmonic intervals to the distances of the stars), continued. The First Opinion, of the Pythagoreans, is examined: its prior form, giving the ancient distance-estimates in stadia and Earth-semidiameters, is shown to be far too small against observed parallaxes; its posterior musical form, from Pliny and Censorinus, assigns tone-intervals between the spheres summing to six tones, the octave, tabulated with the resulting absurd parallaxes. Then the Second Opinion, of the Platonists, begins, with Macrobius on the World-Soul woven from music.)

CHAPTER VIII

(continued — the First Opinion (Pythagorean) and the Second (Platonist))

…[which Pierre Dupuy gave to Fr. Marin Mersenne to use, but] which seems to have been **filched from him**. But since we do not know the opinion of these [men], we shall pass down to the views of others.

[Margin: The 1st Opinion — of the Pythagoreans.]

[I.] The **First Opinion** was that of the **Pythagoreans**, which we shall try to fish out from **Pliny**, **Plutarch**, and **Censorinus**. Pliny, then (bk. 2, ch. 21), says: *"Many have attempted to track out the **intervals of the stars** from the earth, and have declared the **Sun to be distant from the Moon nineteen times** as far as the Moon itself is from the earth. But **Pythagoras**, a man of shrewd mind, reckoned it to be from the earth to the Moon about **126,000 stadia**; from her to the Sun double that; and thence to the twelve signs [of the Zodiac] triple"* — in which opinion was also our **Gallus Sulpicius**. Twin to this passage, in part, is that of **Plutarch** (*On the Opinions of the Philosophers*, bk. 2, ch. 31), where he says that **Empedocles** [held] the Moon to be twice as distant from the Sun as from the earth; the **Mathematicians**, that the Moon is distant from the Sun nineteen times as much as from the earth itself; and **Eratosthenes**, that the Moon is distant from the earth **780,000 stadia**.

[Margin: The distance of the Moon from the Earth and from the Sun, according to the ancient Mathematicians.]

I should believe these Mathematicians — who determined the distance of the Moon from the Sun to be **nineteenfold** the distance of the Moon from the earth — to be **Aristarchus** and **Hipparchus** and their followers. For Aristarchus, in his book *On the Magnitudes and Distances of the three bodies* (proposition 7), had demonstrated that the distance at which the Sun is from the earth is **greater than eighteenfold** the lunar distance from the earth, but **less than twentyfold** — that is, about nineteenfold. And **Hipparchus** [put] the Moon's distance at about **64 terrestrial semidiameters**, and the Sun's at **1200**, which to 64 is about as **19 to 1**.

It is therefore astonishing that **Eratosthenes** assigned to the lunar distance only **780,000 stadia**, that is, **97,500 Italian miles**. For (as said in bk. 2, ch. 7) Eratosthenes set the diameter of the Earth at **80,181 stadia**, and its semidiameter at **40,090½**; and 780,000 stadia divided by 40,090½ yield, for the lunar distance, a full **19 terrestrial semidiameters**. But much more shamefully must **Pythagoras** have blundered, who allowed this distance no more than **126,000 stadia**, which divided by 8 make **15,750 Italian miles** — that is, scarcely **4 terrestrial semidiameters** — though the **lunar parallax** protests, from which we already derive a distance from the earth of more than **51 semidiameters** (bk. 4, ch. 14). And that distance doubled would make the Sun's distance **8 terrestrial semidiameters**, and tripled, the distance of the Fixed stars **12 semidiameters** of the Earth; so that, if we followed the ancient Pythagorean opinion, the **Sun and the Fixed stars would lie under a parallax of several degrees**, against the evidence of observations. These rudiments being set down, let us pass to the other opinion of his.

[Margin: The later opinion of Pythagoras.]

[II.] Having finished ch. 21, then, **Pliny** (bk. 2, ch. 22) subjoins, saying: *"But **Pythagoras** sometimes, by a musical reckoning, calls a **tone** the distance by which the Moon is from the earth; from her to **Mercury** half of that; and from him to **Venus** nearly the same; from her to the **Sun** a sesquiple [a tone and a half]. From the Sun to **Mars** a tone — that is, as much as from the earth to the Moon; from him to **Jupiter** half; and from him to **Saturn** half; and thence a sesquiple to the **Zodiac**: so that **seven tones** are made, which they call the **Diapason** harmony — that is, the universality of concord. In it Saturn is moved in the **Dorian**, Mercury in the **phthongus**, Jupiter in the **Phrygian**,"* and the like in the rest — with a subtlety **more pleasant than necessary**.

Which opinion **Censorinus** (*On the Birthday*, ch. 11) hands down indeed more explicitly, but in the distance of Saturn from the Fixed stars he differs from Pliny (unless the text be corrected); for he says: *"Pythagoras taught that **this whole world was made by a musical reckoning**, and that the seven stars wandering between heaven and earth — which govern the births of mortals — have a **rhythmic** motion (ἔρρυθμον), and intervals*

***agreeing with the musical diastemata**, and render various sounds according to their several heights, so concordant that they sing a most sweet melody, yet one inaudible to us because of the magnitude of the sound, which the narrowness of our ears could not take in."* And a little after: *"Pythagoras judged how many stadia there were between the earth and the several stars. (By a stadium, in this measure of the world, is chiefly to be understood what they call the Italian, of 625 feet — that is 125 paces — which make the eighth part of an Italian mile.)"* Then he proceeds, saying: *"Therefore from the earth to the Moon he thought there were about **126,000 stadia**, and that this is the interval of a **tone**; from the Moon to the star of **Mercury** (which is called **Stilbon**, στίλβων), half of that, as it were a **hemitone** (ἡμιτόνιον); thence to **Phosphoros** (φωσφόρον), which is the star of **Venus**, nearly the same — that is, another hemitone; from there onward*

*to the **Sun**, three times as much, as it were a tone and a half: so that the star of the Sun is distant from the earth **three tones and a half**, which is called a **Diapente** [a fifth]; but from the Moon, two and a half, which is called a **Diatessaron** [a fourth]. From the Sun to the star of **Mars** (whose name is **Pyroeis**, πυρόεις) there is as much interval as from the earth to the Moon, and this makes a **tone** (τόνον); thence to the star of **Jupiter** (which is called **Phaethon**, φαέθων), half of it, which makes a **hemitone**; and as much again from Jupiter to the star of **Saturn** (whose name is **Phainon**, φαίνων), that is another hemitone; thence to the highest heaven, where the Signs are, [another] hemitone. So that from the highest heaven to the Sun the interval is a **Diatessaron** — that is of two tones and a half — but from the heaven's summit to the earth there are **six tones**, in which is the **Diapason** (διὰ πασῶν) symphony."*

The numbers and proportions of Censorinus **Zarlino** too described (*Harmonic Institutions*, part 1, ch. 6), setting from the earth to the supreme heaven — not from his own, but from Pythagoras's opinion — **six tones**. For he himself denies that from the earth to the Moon there is a tone, since the earth, being **immobile**, is unfit to represent any sound, and so to make music with the Moon. **Georgius Valla** (*Music*, bk. 1, ch. 2) refers that "sesquiple" in Pliny not to a tone but to a **semitone**, so that from Saturn to the Fixed stars there are **three-quarters of a tone**. But **Glareanus** (*Dodecachordon* bk. 2, ch. 3) affirms that all the old codices of Pliny have not [the sesquiple] but **six tones**, and so that for "sesquiple" one should read **"semitone"**; and he denies that the **Diapason** can arise, by any musicians' opinion (much less by Aristoxenus's), out of **seven** tones.

[Margin: Pliny must be corrected.]

Certainly, if we consult the scale of **Guido** taken from Boethius (but augmented and corrected), which we set out in ch. 6, num. 3, we shall see that from the first string to the octave (which make the consonance **Diapason**) there are indeed **seven intervals**, but only **six tones** — namely five whole [tones] and two hemitones. Finally, the most diligent author **Censorinus** cannot otherwise be reconciled with Pliny. Since he affirms that the **Pythagorean stadia are equal to the Italian**, and so puts from the Moon to the Earth only **126,000 stadia** — that is **15,750 Italian miles**, which are about **4 terrestrial semidiameters** — we shall easily construct the following little table, with the **parallaxes agreeing** with the Pythagorean intervals.

Pythagorean Intervals

(Pythagorica Intervalla, ex Plinio correcto & Censorino — "from corrected Pliny and from Censorinus")

Body	Distance from Earth (Earth-semidiam.)	Interval to next body	(semidiam.)	Horizontal parallax
Earth (*Tellus*)	0	Tone (*Tonus*)	4	—
Moon	4	Semitone	2	14°28′
Mercury	6	Semitone	2	—
Venus	8	Sesquitone (*Sesquitonus*)	6	—

Body	Distance from Earth (Earth-semidiam.)	Interval to next body	(semidiam.)	Horizontal parallax
Sun	14	Tone (*Tonus*)	4	4°6′
Mars	18	Semitone	2	—
Jupiter	20	Semitone	2	—
Saturn	22	Semitone	2	—
Fixed stars	24	—	—	2°23′

*(The original sets three columns: "Distances of the planets among themselves" (the interval-name and its value in Earth-semidiameters — Tone = 4, Semitone = 2, Sesquitone = 6), "And from the Earth" (the cumulative distance in semidiameters), and "Horizontal Parallax" (in degrees and minutes). The intervals sum to **6 tones = the octave**; the distances run 4-6-8-14-18-20-22-24 Earth-semidiameters. The parallaxes are printed only for the Moon, Sun, and Fixed stars; each checks as the arcsine of 1/distance — arcsin(1/4) ~ 14°28', arcsin(1/14) ~ 4°6', arcsin(1/24) ~ 2°23' — and their enormity (whole degrees for the Sun and Fixed stars) is exactly Riccioli's point that the Pythagorean distances are far too small.)*

[Margin: The 2nd Opinion — of the Platonists.]

[III.] The **Second Opinion** was rather of the **Platonists** than of Plato himself, who rejected the planet-distances determined by **Archimedes** for no other cause than that they did not keep the **proportions due to Music** — as **Macrobius** relates (bk. 2 on the Dream of Scipio, ch. 3), where he has it thus: *"Rightly, therefore, is **everything that lives captivated by music**; for the **celestial soul**, by which the universe is animated, took its origin from music. This [soul], while it impels the world's body to its spherical motion, makes a sound — distinguished by unequal intervals, yet marked off by a proportionate reckoning, just as [the soul] was from the beginning woven together."* But these intervals — which in the soul, as incorporeal, are reckoned by **reason alone, not by sense** — must be sought …

*[The text breaks off; the catchword **"Vtrum"** ("Whether…") points to p. 527 (PDF 562), continuing ¶III on the Platonists' opinion, within Chapter VIII.]*

(printed p. 527 — Chapter VIII continued. The Second Opinion (the Platonists) concludes: the Timaeus multipliers yield a table of "Platonic Intervals" for the planetary distances, which Riccioli refutes as wildly at odds with demonstrated solar and Saturnian distances — "farewell to this Pythagorean and Platonic Harmony." The Third Opinion, of Fr. Mario Bettini, then begins: the world-radius as a monochord of 3435 semidiameters, with the Sun at 1145 sounding "Sol" and the planets placed at octave consonances of the whole chord; the page ends with the head of Bettini's monochord table.)

CHAPTER VIII

(continued — the Platonists' opinion, and Bettini's)

[Margin: The absurdity of the Platonic intervals.]

…[these intervals must be sought by reason, not by sense.] And **Archimedes** indeed believed he had grasped the number of stadia by which the **Moon** is distant from the earth's surface; the **Moon** from Mercury; Mercury from **Venus**; the **Sun** from Venus; **Mars** from the Sun; **Jupiter** from Mars; **Saturn** from Jupiter; and from Saturn's orb up to the starry heaven itself, he thought he had measured out the whole space by reason. Yet this Archimedean measurement was **rejected by the Platonists**, as not keeping the **double and triple intervals**. And they laid it down that this must be believed: that, as great as is [the distance] from the earth to the **Moon**, double of that is from the earth to the **Sun**; and as great as is from earth to the Sun, **triple** of that is from earth to **Venus**; and as great as from earth to Venus, **fourfold** is from earth to the star of

Mercury; and as great as is from earth to Mercury, **ninefold** is from earth to **Mars**; and as great as is from earth to Mars, **eightfold** is from earth [to Jupiter; and as great as from earth to Jupiter,] **twenty-sevenfold** is from earth to the orb of **Saturn**.

This Platonic persuasion **Porphyry** inserted in his books, by which he infused not a little light into the obscurities of the **Timaeus**; and he says that they believed the intervals in the body of the world to be filled, after the image of the soul's weaving, by **Epitrites** [4 : 3], **Hemiolia** [3 : 2], **Epogdoa** [9 : 8], and **Hemitones and Limmas**, and that thus the concord arises. Which proportions **Marsilio Ficino** also reports (in his compendium on Plato's *Timaeus*, ch. 34), and asserts that they seem to him more probable, and according to Plato's mind in the *Timaeus* and in *Republic* 8 and 10. That we may weigh these intervals, then, we shall assume the lunar distance from the Earth to be nearest to **60 terrestrial semidiameters** (per what was said in bk. 4, ch. 4); which posited, the **Platonic Intervals** will be as may be seen in the following table.

Platonic Intervals

(Intervalla Platonica)

Body	Proportion to the earth-distance of the body just preceding	Distance from Earth (the Lunar being 1)	Distance from Earth (the Lunar being 60, i.e. Earth-semidiameters)
Moon	(the unit)	1	60
Sun	double the Lunar	2	120
Venus	triple the Solar	6	360
Mercury	quadruple the Venereal	24	1440
Mars	ninefold the Mercurial	216	12960
Jupiter	eightfold the Martial	1728	103680
Saturn	twenty-sevenfold the Jovial	46656	2799360

(Verified: each "distance from Earth" is the running product of the proportions — 1, 1·2, 2·3, 6·4, 24·9, 216·8, 1728·27 = 1, 2, 6, 24, 216, 1728, 46656; times 60 = the last column. The multipliers 1, 2, 3, 4, 9, 8, 27 are Plato's Timaeus soul-numbers.)

There would be, therefore, a distance of the **Sun from the Earth of only 120 terrestrial semidiameters** — which conflicts with what we demonstrated (bk. 3, ch. 7); but a distance of **Saturn from the Earth of about 2,799,360 terrestrial semidiameters** — that is, to the solar distance as **23,328 to 1**, whereas it ought to be only about **10 to 1**, or nearly tenfold, by what was said (bk. 7, sect. 6, ch. 1 and 2). Farewell, then, to this **Pythagorean and Platonic Harmony**, which does not reconcile reason with the experiments of the senses and with observations — though it ought to have been reconciled, as **Ptolemy** rightly decides (*Harmonics* bk. 1, ch. 1 and 2). For it is not enough that these proportions agree with reason, even if they cannot be approved by sense — as **Zarlino** seems to have admitted (part 1, ch. 6).

[Margin: The 3rd Opinion — Bettini's.]

[IV.] The **Third Opinion** is that of our **Fr. Mario Bettini** (*Apiarium* 10, Progymnasma 1, propositions 1 and 3, to be read with their Scholia). He supposes, first, that the **Sun is distant from the center of the Earth 1145 terrestrial semidiameters**, and is in the **middle of the Planetary System**; and that its distance, taken with the remaining distance up to the **Empyrean**, makes a **Diapente** [a fifth]; and therefore that the Sun's distance from the Earth must be **tripled**, so that the semidiameter of the whole World may be held as one entire **Monochord** — namely **3435 terrestrial semidiameters**. Whence it follows that, ascending from the earth to the Sun, the **Sun occupies the place of the fifth string**, and to it agrees the voice which in Guido's musical Scale is called **Sol** — fitting to the name of the Sun (*Sol*).

Then he determines the distances of the planets — indeed, of the heavens above the planets too — according to the consonances of the more recent Musicians, admitted in the common use of the **Octochord**; but an Octochord **purely Dorian**, and holding the **Diatonic** gravity. Further, that he may serve both the ease of Arithmetical operation and the **order of the planets** found by the more recent Astronomers, and that at the same time the harmonic consonances may **disagree as little as possible with observations**, he takes some planets at their **mean** distance, some at their **Apogee**, some at their **Perigee**. These being supposed, he determines the intervals of the planets and heavens from the earth, in terrestrial semidiameters, as you see in the following table — in which, as I said, the distance of the **Empyrean** from the earth's center, that is, the whole chord, is **3435** terrestrial semidiameters; and:

- the **Moon at Apogee** — so that by the ancients it be a **major Tone**, which is the ninth part of the whole chord — is **381 6/9** semidiameters;
- the **Venus at Perigee** makes the consonance of a **Ditone** [major third], namely the fifth part of the whole chord, which is **687** semidiameters (so that between the Moon at apogee and Venus at perigee he sets a **minor tone**);
- the **Mercury at Perigee** makes the consonance of a **Diatessaron** [fourth], which is the fourth part of the whole chord, **858 3/4** semidiameters (so that between Mercury and Venus, both at perigee, he sets a **major semitone**, or **Apotome**);
- the **Sun's mean** distance, as said, **1145** semidiameters, with the residue of the whole chord (which is **2290**) makes a **Diapente** [fifth], and with the whole chord a **Diapason-diapente** [twelfth]; whence between Mercury and the Sun falls a **major Tone**, as he supposes;
- the **Mars mean** distance occupies the terminus of the **major Hexachord** [major sixth], that is two-fifths of the whole chord, and so **1374** semidiameters;
- to hunt out **Jupiter's** distance, he subtracts the major hexachord (**1374**) from the whole chord (**3435**), and there remain **2061**; of this he takes the ninth part as a **major tone**, namely **229**, and adds it to Mars's distance, and Jupiter's distance from the earth becomes **1603** semidiameters;
- **Saturn's** distance he makes **half** of the whole mundane semidiameter, or of the whole chord, that is **1717 1/2** semidiameters, that it may acquire the consonance of a **Diapason** [octave] with the whole chord;
- the **Fixed stars'** distance he makes **two-thirds** of the whole chord [**2290**], that they too may sound a **Diapente**;
- the **Crystalline heaven's** distance he determines by the consonance **Diatessaron**, but so that from earth to the Crystalline there be three-quarters of the whole chord, that is **2574 3/4** semidiameters;
- whence it comes about that the distance of the **Empyrean and Crystalline** to the whole chord is as **1 to 4**, which is the **Disdiapason** [double octave].

From these we have composed the **following table**, drawn from his own express words, yet with **many consonances added** that arise from his intervals, which he passed over in silence.

Harmonic Intervals — from Fr. Mario Bettini

(Harmonica Intervalla, indicated on the line AB as the entire String of the World-Monochord)

Point on AB	Body	Distance from Earth (Earth-semidiam.)	Proportions, with the consonances or fitting intervals
A	Earth (*Tellus*)	—	—
D	Moon at Apogee	381 6/9	AB to BD, as 9 : 8 (a Tone)

Point on AB	Body	Distance from Earth (Earth-semidiam.)	Proportions, with the consonances or fitting intervals
E	Venus at Perigee	687	AB to BE, as 5 : 4 (a Ditone); AB to AE, as 5 : 1 (Disdiapason-with-Ditone)

*(This table only **begins** here — the line AB is the whole world-monochord, 3435 semidiameters, A being the Earth and B the Empyrean; for each body the proportion of the whole string AB to the outer segment B–[point] gives a consonance. It breaks off after Venus, with the catchword "RESI-"; the remaining bodies — Mercury, Sun, Mars, Jupiter, Saturn, the Fixed stars, the Crystalline, and the Empyrean — follow on p. 528. Note: 3/4 of 3435 is 2576 1/4, so the printed Crystalline value **2574 3/4** appears to be a small slip or rounding; all the other figures check exactly against the 3435 chord.)*

*[The catchword **"RESI-"** (Residuum…) points to p. 528 (PDF 563), which continues Bettini's monochord table, within Chapter VIII.]*

(printed p. 528 — Chapter VIII continued. Bettini's monochord table is completed, then Riccioli critiques his system: though more concinnous than the Pythagorean and Platonic schemes, it contains six errors — impossible lunar and solar distances, incompatibility with lunar and solar eclipses, Jupiter and Saturn placed far too near, and fixed stars so close as to imply a false parallax — all sprung from Bettini's wish that the Sun sound "Sol" in the fifth place. The Fourth Opinion, Kepler's harmonic intervals from Harmonics book 5, then begins with the head of his table of planetary aphelion and perihelion distances.)

CHAPTER VIII

(continued — Bettini's table completed and refuted; then Kepler's opinion)

Remainder of the preceding Table

(Residuum Tabulae Praecedentis — continuing Bettini's world-monochord table from p. 527; the line AB = 3435 Earth-semidiameters, A the Earth, B the Empyrean)

Point on AB	Body	Distance from Earth (Earth-semidiam.)	Proportion (AB to B-point)
F	Mercury at Perigee	858 3/4	AB to BF, as 4 : 3 (Diatessaron)
C	Sun, mean	1145	AB to BC, as 3 : 2 (Diapente)
H	Mars, mean	1374	AB to BH, as 5 : 3 (major Hexachord)
I	Jupiter at Perigee	1603	AB to BI, as 3435 : 1832
G	Saturn at Perigee	1717 1/2	AB to BG, as 2 : 1 (Diapason)
K	Fixed stars	2290	AB to BK, as 3 : 1 (Diapason-with-Diapente)
L	Crystalline	2574 3/4	AB to BL, as 4 : 1 (Disdiapason)
B	Empyrean	3435	(the whole string)

Further consonances arising from the same intervals (which Bettini "passed over in silence"), measuring the whole string AB against each inner segment A–[point], and segment against segment:

Proportion	Ratio	Consonance
AB to AF	4 : 1	Disdiapason
AB to AC	3 : 1	Diapason-diapente
AB to AH	5 : 2	Diapason-with-Ditone
AB to AG	2 : 1	Diapason
AB to AK	3 : 2	Diapente
AB to AL	4 : 3	Diatessaron
AF to AG	1 : 2	Diapason
AK to AC	2 : 1	Diapason
AH to AF	2 : 1	Diapason

(All verified against the 3435 chord, with B-point = 3435 - [distance]: e.g. BF = 2576¼, so AB:BF = 4:3; BG = 1717½, so AB:BG = 2:1; BK = 1145, so AB:BK = 3:1; BL ~ 858¾, so AB:BL = 4:1. The last added line, ***"AH to AF, as 2 to 1, Diapason,"*** *is printed thus but does not check: AH:AF = 1374 : 858¾ = 8:5, a minor sixth — the 2:1 octave actually holds for AH:AE = 1374 : 687, Mars-mean to Venus-perigee; the printed "AF" appears to be a slip for "AE.")*

[Margin: Bettini's. — 1st error.]

[V.] Although the aforesaid symmetry [of Bettini] is **much more concinnous** than either the Pythagorean or the Platonic, yet it includes **many repugnancies and fallacies**, and induces a most absonant **discord between Astronomy and Harmonics**. **For, first**, it attributes to the **Moon** so great a distance as no astronomer ever assigned, or can assign, saving the parallaxes. For no one ever found, or will find, in it a **horizontal parallax greater than 1°43′, or less than 51½′** — of which the former imports a distance from the earth of about **33½ terrestrial semidiameters**, the latter of about **67** — as is certain from Geometry and from what was said (bk. 4, ch. 14). And this very manner of determining the distances of the stars by parallaxes Bettini himself approves (*Apiarium* 8, Progymn. 3, prop. 9; Progymn. 4; and *Aerarium Philosophiae* vol. 2, p. 73). [His lunar distance of 381, then, is impossible.]

[Margin: 2nd error.]

Secondly: from the lunar distance of **381 6/9 semidiameters** which he sets, it would follow that the **Moon could never be eclipsed by the Earth's shadow** — which is against the experience of every age, and against all who are even slightly versed in Astronomy; nay, against what the same Father most learnedly teaches concerning the lunar eclipses made by the Earth's shadow (*Apiarium* 8, progymn. 2 and 3, prop. 11). For **no one ever raised the height of the terrestrial shadow beyond 282 semidiameters** of the Earth (as is clear from bk. 3, ch. 11, Probl. 8). Nor may you say that hence only the eclipse of the Moon **at apogee** is removed, but not below the apogee; for, the 282 semidiameters of the shadow being subtracted (at the most favorable), from the lunar distance of 381 there remain **99 terrestrial semidiameters** — a difference between the

Moon's perigee and apogee that no one assigns; and yet we know that the Moon, even at apogee, has sometimes fallen into a total eclipse, and so far below the apex of the Earth's shadow.

[Margin: 3rd error. — 4th error.]

Thirdly: a total eclipse of the Sun could indeed happen if the Moon were distant from the earth 381 and the Sun 1145 semidiameters, as he sets; for, the apparent diameters of the luminaries being preserved, such an eclipse would be **impossible** (as is clear from bk. 5, ch. 9); yet Bettini himself admits a total solar eclipse (*Apiarium* 8, Progymn. 2, prop. 10), and is bound to admit it from the history of eclipses, of which we [treat] (bk. 5, ch. 20). **Fourthly**: the lunar distance of 381 semidiameters cannot **cohere** with the solar distance of 1145 which he posits, as is proved by **Aristarchus's problem founded on the phase of the lunar dichotomy** [half-moon] — which Bettini himself praises (*Aerarium* vol. 1, p. 629); for, the lunar distance being put at 381 6/9, or nearly 382, it follows that the **Sun is distant from the earth at least 7299 terrestrial semidiameters** — indeed in truth much more, the [dichotomy] angle being posited, which we observed at the time of the dichotomy, as is clear from what was demonstrated (bk. 3, ch. 7, probl. 3).

[Margin: 5th error. — 6th error.]

Fifthly: he places **Jupiter and Saturn far nearer to the Sun and the Earth** than the **prosthaphaereses** [equations of the orbs] and the commensurations of the orbs (demonstrated by Copernicus and others) require; for these require that the **distance of Jupiter be nearly fivefold the distance of the Sun from the earth**, and that of **Saturn nearly tenfold** (as we taught, bk. 7, sect. 1, ch. 1 and 2); whereas by his [scheme] Jupiter's distance to the Sun's distance is **less than 14 to 11**, and Saturn's **less than 16 to 11**. **Sixthly**: he gives the **[Fixed] stars** a distance from the earth of **3435** — so small that no astronomer could ever assign it, since from it would follow a **horizontal parallax of a minute and a half**, which all astronomers know to be false.

[Margin: Other incongruities.]

Now **all these errors arose** from this: that he wished the Sun's distance both to be in the **fifth place** and to utter the voice **"Sol,"** and to make a **Diapente** from the supreme heaven with the whole semidiameter of the World; and [that he wished] the Moon to make a **tone**, that is, to contain one-ninth of that whole interval in its distance, and so forth. I pass over that he assigns to **Jupiter** — not the least of the planets — **no worthy consonance**; that he makes a certain consonance answer to four planets **outside their perigee position**, to the Moon outside apogee, to the Sun and Mars outside their mean distance — as though indeed they did not usually keep the harmony; which is laid down too inconsistently by one who has taken upon himself the demonstration of the **Harmonic proportions observed by God in the intervals of the planets**. Finally, that the Sun may hold the place of the fifth string, the **Earth** is assumed by him as the **first string** — though both [the Earth] and the **Empyrean**, by their immobility, are **unfit to represent a sound**.

[Margin: The 4th Opinion.]

[VI.] The **Fourth Opinion** is that of **Johannes Kepler** (*Harmonics* bk. 5, **ch. 4**), where he **sets out the extreme intervals from Tycho's observations** — that is, the **distances of the planets from the Sun** — in which, at the aphelia and perihelia, the **harmonies are found**, except for **Mars and Mercury**, as appears from the Table which he exhibits, as here below. For the understanding of which it must be supposed (from ch. 3 of the same book) that the **extreme Convergence** of two planets is when their apsides are nearest — namely at the **Perihelion of the superior** and the **Aphelion of the inferior** planet; but the **extreme Divergence** is the **opposite** apsides of the two planets — namely the **Aphelion of the superior** and the **Perihelion of the inferior**. Let the Table now stand:

Intervals compared with the Harmonics

(Intervalla Comparata cum Harmonicis — of which the radius of the Annual Orb [the Earth's orbit] is 1000)

Planet	Apsis	Distance (annual-orb radius = 1000)		The proportion of each [planet's own interval]
Saturn	Aphelion	10052	a	More than a minor tone — 10000 : 9005
	Perihelion	8968	b	Less than a major tone — 10000 : 8911
Jupiter	Aphelion	5451	c	No fitting proportion, but nearly as
	Perihelion	4949	d	11 to 10, or half of 6 : 5
Mars	Aphelion	1665	e	If it were 1665 : 1385, the harmony would be 6 : 5
	Perihelion	1382	f	If it were 1665 : 1332, the harmony would be 5 : 4

*(This table only **begins** here — it sets each planet's distance from the Sun at aphelion and perihelion, the Earth's orbital radius being 1000, and compares each planet's own aphelion-to-perihelion interval with a musical consonance. Saturn's interval, 10052 : 8968 = 1.121, lies between a minor tone (10:9) and a major tone (9:8); Jupiter's, 5451 : 4949 ~ 11:10, fits no clean consonance; Mars's, 1665 : 1382 ~ 1.205, is near 6:5. It breaks off after Mars, with the catchword "RESI-"; the remaining bodies — Earth, Venus, Mercury — and the comparison columns follow on p. 529.)*

*[The catchword **"RESI-"** (Residuum…) points to p. 529 (PDF 564), which continues Kepler's table, within Chapter VIII.]*

(printed p. 529 — Chapter VIII continued. Kepler's table of intervals is completed, and its use explained: single planets' own aphelion-perihelion intervals yield no clean harmony, but the convergent and divergent intervals between adjacent planets do — yet Kepler himself denies that harmonic proportions are to be sought among mere lengths, harmony's true subject being the motions. The Fifth and Riccioli's own Opinion then begins: the distances of planets and fixed stars are not determined by harmonic proportions, supported by Aristotle, Pliny, Macrobius, Zarlino, Glareanus, Mersenne, and Kircher's doctrine of a harmony of disposition rather than sound.)

CHAPTER VIII

(continued — Kepler's planetary harmonies, and the Fifth (Riccioli's own) Opinion)

Remainder of the preceding Table

(Residuum Tabulae Praecedentis — completing Kepler's "Intervals compared with the Harmonics" from p. 528; annual-orb radius = 1000)

Planet	Apsis	Distance (annual-orb radius = 1000)		The proportion of each [planet's own interval]
Earth	Aphelion	1018	g	If it were 1020 : 980, it would be a Diesis (25 : 24);
	Perihelion	982	h	it does not, therefore, possess a Diesis

Planet	Apsis	Distance (annual-orb radius = 1000)		The proportion of each [planet's own interval]
Venus	Aphelion	729	i	Less than a sesquicomma;
	Perihelion	719	k	more than a third part of a Diesis
Mercury	Aphelion	470	l	More than a Diapente, abounding;
	Perihelion	307	m	243 : 160, less than the harmonic 8 : 5

(The full index-key for both halves of Kepler's table — a = Saturn aphelion (10052), b = Saturn perihelion (8968); c = Jupiter aph. (5451), d = Jup. per. (4949); e = Mars aph. (1665), f = Mars per. (1382); g = Earth aph. (1018), h = Earth per. (982); i = Venus aph. (729), k = Venus per. (719); l = Mercury aph. (470), m = Mercury per. (307).)

[Margin: Use of the table.]

The **extreme intervals of no single planet**, then, allude to Harmonies — except those of **Mars and Mercury**. For in **Mars**, if its perihelion 1382 were 1388, it would be to its own aphelion 1665 as **5 to 6**, and so would be the consonance of a **semiditone** (minor third) — from which, however, it is little distant. But in **Mercury**, although to Kepler it seems to allude to a **Diapente**, or to the harmony **8 : 5** (the minor Hexachord), to me its extreme intervals seem rather to allude to a **Diatessaron** (which is between 4 and 3) — or else this interval is to be despised [as negligible].

But **if you compare the extreme intervals of different planets among themselves** — says Kepler — **some light of Harmony shines forth**, as is clear to one who contemplates the first column of the preceding table and its index-characters of the extremes:

Adjacent pair	Extreme **Divergence** (aphelion of upper : perihelion of lower)	Extreme **Convergence** (perihelion of upper : aphelion of lower)
Saturn–Jupiter	**a : d** = 10052 : 4949, ~2 : 1 (a little more than a **Diapason**)	**b : c** = 8968 : 5451 — between 5:3 and 8:5 (between the **major and minor Sixth**)
Jupiter–Mars	**c : f**, ~4 : 1 (nearly a **Disdiapason**)	**d : e**, ~3 : 1 (nearly a **Diapason-with-Diapente** [twelfth])
Earth–Mars	**e : h** — somewhat more than 5 : 3 (the **major Sixth**)	**f : g** — an **abounding Diatessaron** (more than 4 : 3)
Earth–Venus	—	**g : k** — an **abounding Diatessaron**
Venus–Mercury	**i : m** — a little less than 12 : 5 (a **Diapason-with-semiditone** [octave + minor third])	**k : l** — a little more than 3 : 2 (a **Diapente**)

But Kepler subjoins that these intervals, in so far as they are **lengths without motion**, are not aptly examined for Harmonies — whose subject is rather **motion itself**, as to its swiftness and slowness; and therefore, if we seek harmonies, they are not to be sought in the intervals in so far as they are **semidiameters of the orbs**, but in so far as they are the **measure of the motions** — that is, rather in the motions themselves; especially since, for the semidiameters of the orbs, nothing can be taken but the **mean** distances from the Sun, among which the Harmony shines forth **less** than in the extremes of the aphelia and perihelia. Wherefore Kepler

absolutely denies that harmonic proportions are to be sought among the **intervals** of the planets as such, and apart from motion.

The Fifth and Our Opinion, and the Authorities for it

(Quinta et Nostra Opinio, et Authoritates pro illa)

> *[Margin: Aristotle's. — Macrobius's. — Zarlino's, Glareanus's. — Mersenne's. — Fr. Athanasius's praise and opinion in matters of Music.]*

[VII.] The **Fifth — and Our — Opinion**, not unlike the preceding and truer, is that the **distances of the Planets and Fixed stars** — whether from the Earth, or from the Sun, or among themselves — are **not to be determined by Harmonic proportions or Musical intervals**. Of this opinion, without doubt, was **Aristotle** (*On the Heaven* 2, text 52), where, of the **Pythagoreans** who sought a harmony in the motion and intervals of the heavens, he said that *"this is indeed said wittily and elegantly, yet the truth is not so."* And **Pliny** (bk. 2, ch. 22), where, after relating Pythagoras's opinion of the intervals of the stars by Tones, Semitones, and Sesquitones, he concludes that these and the like said by him were *"with a subtlety pleasant more than necessary."* Of the same opinion too was **Macrobius** (bk. 2 on the Dream of Scipio, ch. 4), thinking this inquiry belongs to one *"showing off, not teaching"* — whose opinions we have so often inculcated, as often as occasion returns to us.

To Aristotle and Pliny subscribed in this **Zarlino** (*Institutions* part 2, ch. 29) and **Glareanus** (*Dodecachordon* bk. 1, ch. 13, and bk. 2, ch. 13), where he says: *"But whether the intervals of the orbs, in the heaven itself, stand by the same reckoning as the phthongi [notes] in the Diapason, does not seem to me probable, by whatever genus of mode we may finally constitute them."* And, some things being inserted, [he adds that] **Boethius**, the true judge of this business — since he saw these things wonderfully varied among the ancients, and Pliny not afraid of a subtlety pleasant more than necessary — so **tempered** [the matter] as nevertheless to set both opinions before the eyes; and finally, declaring his own opinion, [says] that *"not without reason does this notion seem to Aristotle pleasant to say rather than likely to be true."*

Nor is there doubt that **Mersenne** (on Genesis 4, verse 21, p. 1558) was of the same opinion, when he says: *"I know it is not necessary that the mutual distance of the Planets among themselves and with the Earth be exactly [proportioned] to their sounds and magnitudes, so that their Music may represent their economy [order] — though would that it could be done to the line, that we might have some commerce with [their] voices and instruments."* And (p. 1559) he makes light of **those very allusions and consonances which we have reported from Kepler** in the preceding number, because the opinions of Astronomers about the distances of the stars are **diverse** — which [of them] the Musician should choose [he cannot tell]. Again (p. 1703), where he reports the opinion of certain [astrologers] saying that the **Diapason and Disdiapason are governed by the Sun, the Diapente by Venus, the Diatessaron by [Mars], and the Diapason-diapente by Jupiter**, he says (on the following page) that all these are to be repudiated, and adds: *"In vain, therefore, will there be harmonists, if [they hold it to be] from the earth to the Moon a tone; from the Moon to Mercury a semitone, and from him to Venus as much; from the earth to the Sun a Diapente; from the Moon to the Sun a Diatessaron; from the Sun to Mars a tone, thence to Jupiter a semitone, as much from Jupiter to Saturn, and from Saturn to the Firmament; and consequently from the Sun to the Firmament a Diatessaron, and from the earth to the Firmament a Diapason."* For it is sufficiently clear, by Geometric and Astronomical experience, that **those spaces are wrongly constituted** (as appears from article 3, and from many other places, concerning the distance of the heavens from one another); but the conclusion must be drawn **from the proportion of the motions alone**.

Most recently, **Fr. Athanasius Kircher** treats best of all of the **Music reigning in the heaven and in all Nature** — especially in *Musurgia* bk. 10, which he entitles the *Decachord of Nature* (Register 2, paragraph

2); inquiring what the harmony of the celestial bodies is, and in what it consists, he concludes that their **Harmonic consent** consists *"not in the periodic numbers of the motions, nor in any sensible collision of the celestial bodies, but in nothing else than in their wonderful **disposition**, and a certain ineffable proportion, conspiring into unity — by which the mundane bodies so correspond to one another that, one being removed or changed, the harmony of the whole would deservedly perish,"* and so on. Which harmony, as we have said, consists in the wonderful disposition and most proportionate [arrangement] of one body …

> *[The catchword **"ris"** (corpo·**ris**) points to p. 530 (PDF 565), which continues Kircher's view and Riccioli's conclusion, within Chapter VIII.]*

(printed p. 530 — Chapter VIII continued. The Fifth Opinion's Kircherian thread concludes: the true celestial harmony is God's providential disposition of distances and magnitudes for the good of sublunary nature, not any audible sound. Five reasons for Riccioli's opinion follow: the motions make no sensible sound; any consonances would exceed the range of human music; harmonic ratios of magnitude are not true harmony, which lies in the quality of sounds; the harmonics cannot be reliably located among the disputed distances; and harmony's arbiters are sense and reason together, per Ptolemy.)

CHAPTER VIII

(continued — the Fifth (Riccioli's own) Opinion, and the Reasons for it)

…[the harmony consists in the wonderful disposition and most proportionate analogy] of one **mundane body** to another, by their interval; and also in the **most exactly fitted analogy of the quantity, or magnitude, of each** for obtaining its own end. But the **end intended by God was not the delight, sensible to hearing, of a sound roused from the heavens**, but the **production, conservation, and advancement of sublunary nature** (especially of living things) toward [its] ultimate end, each in its own manner — which **Kircher** there goes on to expound excellently and at length. And therefore [God] attributed **so great a distance to the Planets** that they might produce effects agreeing with sublunary natures: for if the **Moon or Sun** were much nearer than they are, the former would moisten and chill [things] too much, the latter would dry up and overheat them; and unless, between **Saturn and Mars** — those discordant, pestiferous, and malign planets — He had interposed **Jupiter with its four companions** [the four moons], wholesome and most temperate, and on this side the beneficent **Sun and Venus**, great and intolerable harms would have followed.

This, before Kircher, **Pliny** had noted (bk. 2, ch. 8), where, having said that the **star of Saturn** is *"of a cold and stiffening nature,"* and a little after that the **third star, of Mars** (which some call Hercules's) is *"fiery and burning from the nearness of the Sun, completing [its course] in about two years,"* he at once subjoins: *"And therefore, interposed between both by the excessive heat of the one and the cold of Saturn, **Jupiter** is tempered by both, and made wholesome."* And the Ciceronian **Paulus** intimated the same in the Dream of Scipio, when he says: *"of which [orbs] one globe is held by that [star] which on earth they call Saturn's. Next is that prosperous and wholesome gleam to the human race which is called Jupiter's. Then the ruddy one, dreadful to the lands, which you call Mars's."* But what we **stammer** out concerning these things, toward indicating a certain specimen of divine **Providence**, must be understood (if it can be understood by us at all) of **innumerable other reasons and proportions chosen by God** in ordering [all things] to their end.

Reasons for the Fifth and Our Opinion

(Rationes pro Quinta et Nostra Opinione)

[Margin: 1st reason.]

[VIII.] First: the **intervals and motions of the celestial bodies** either produce **no sound at all** — especially in the Planetary system, where the planets move in a **fluid and most tenuous ether** — or at any rate produce no sound **sensible to us**, as is clear by experience; wherefore it was truly said by **Pliny** (bk. 2, ch. 3): *"To us who dwell within, the world glides on silently, alike by day and night."* They are therefore **not ordained by God for sound**; and accordingly the **harmonic proportions are not to be sought in them** — those proportions which, in human voices, are consonant to our ears so as to produce a concinnity pleasant to them; for, **the end being taken away, the order and proportion due to that end is taken away** too. And much more [is this so] in the **intervals themselves, taken apart from motion** — since, as Kepler rightly said, the proper **subject of harmony is not immobile quantity, but mobile**: that is, the very motion of the bodies.

[Margin: 2nd reason.]

Secondly: even if the celestial bodies did, by their motion, produce a sound sensible to us, yet the **consonances of their sounds would not fall within those terms and limits** within which the consonances of human voices fall — which, on account of our **weakness** or the necessity of [our] nature, we confine within the bounds of the **Boethian system or the Guidonian scale**. For since the **lung, throat, palate, tongue, teeth, and lips** are not ordained to this end **only** (that, by forming the voice, we may modulate sweetly), but also to many other ends, these organs had to be so framed that they could descend only to a **certain limit of gravity**, or ascend [only] to [a limit] of acuteness. But in other animals there are those that can form a **graver or acuter voice** than we; and in the strings and pipes of organs far more and subtler differences can be found — how much more in the **motions of the stars**, mingled with such great variety. **In vain, therefore, would we measure the laws of that [celestial] concord by the over-short and meager norm of our [human] harmony**; it would be just as if we wished to **cramp the Angelic songs** — in the innumerable bodies which [the angels], by God's permission, could assume — to our [human] measures.

[Margin: 3rd reason.]

Thirdly: If a harmonic proportion is to be sought in the celestial intervals or motions — chiefly because in them some **symmetry** can be found, or even a double, triple, quadruple, quintuple proportion, etc. (if not most exactly, at least with fractions little vitiating that proportion) — yet, the aforesaid proportions being **granted** (though not yet conceded), it still does not follow from them, as such, that a **true Harmony** is found, but [only such as harmonies] are **in the qualities of voices and sounds**. For so **Panaetius** said (in his book *On the Principles of Geometry and Music*): *"The faculty of consonances is regarded not in the **magnitudes** of voices, but in their **qualities**."* Otherwise, in whatever discrete or continuous quantities are found (reduced by division to discrete ones, in which, besides unity, are the numbers **2, 3, 4, 5, 6**, by which all the **fourteen consonances** reckoned in the first table of ch. 4 are contained), there Harmony would "reign," taken properly or quasi-properly — and so very many **arts would be confounded** with one another. Nor must **every symmetry** be weighed by the **Canon of the musical Monochord**, or by the norm of harmonic ratios. For who would demand this of the **architect** in his buildings, or of the **apothecary** in compounding medicines, or of the **general** in drawing up a battle-line, or of the **shipwright** in building ships, or of **God Himself and Nature** in the structure of the human body, or of animals or plants? It would surely be **ridiculous** to require that the **nerves, intestines, and tendons** should have, in the animal body, that measure which the harmonic proportions require; or that the **five principal vessels (instruments of the soul)** — the **brain, heart, liver, kidneys, and spleen** — should be so commensurate among themselves, as to quantity or motion, that the brain (for example) should have with another member a **Diapason** [octave], the heart a **Diapente** [fifth], the liver a **Diatessaron** [fourth], the kidneys a **Diapason-diapente** [twelfth], the spleen a **Disdiapason** [double octave]. For such a proportion would be **unfit for the end** to which they are ordained — which is far different from the **delight of the ears**. The same judgement, therefore, holds of [the proportions] in the celestial intervals and motions, [as] not having that **symmetry** which is proper to Music

— that is, **symphonism**. Nor is there doubt that **many proportions [are] unfit for begetting consonances** — such as those arising from the number **7** (and 4) compared with others — and yet are **most fit for other effects** intended by God and Nature in the heaven and in other bodies.

[Margin: 4th reason.]

Fourthly: Either the harmonic proportions are to be sought in **whatsoever** intervals of the Planets — and not this, for most intervals do not have them, and so the work of the **Divine Tuner** [Harmost] would for the most part **lack the very harmony** which the followers of the Pythagoreans so greatly commend; or [they are to be sought] from the **more notable** intervals only — namely from the **Maximum, Mean, and Minimum** distance of the stars among themselves, or from the Sun, or from the Earth, or from [combinations of] all these — and **neither can this be obtained**: both because the **diversity of opinions** about these stellar intervals among the more recent Astronomers stands in the way; and because, not even in any single Astronomer's scheme, are all the maximum, mean, or minimum intervals of the planets so attuned that the **commonly received musical consonances** are in them; and, finally, because **never have all the planets' Apogees concurred with [other] Apogees, or [their] Perigees with Perigees, in one place** under the Fixed stars — or if ever this was [or will be], it does not suffice for a **Harmony worthy of God**, for that ought to happen **always, or for the most part**. Just as it does not suffice for the excellence of some **chief-musician** if the voices and tones are so ordered by him that they **consonate twice or thrice but dissonate a thousand times**. Not to mention, meanwhile, the **variety of the Genera and Modes of Music** (indicated in ch. 5), which is so great that **it is not certain which Genus is suited to the celestial [bodies]**.

[Margin: 5th reason.]

Fifthly: Finally, no one is so senseless, or of so stubborn or shameless a brow, as not to grant to **Ptolemy** (*Harmonics* bk. 1, ch. 1 and 2) that the **arbiters of harmony ought to be Sense and Reason** — yet so that **Sense**, a posteriori, first finds what is **near to the truth**, while **Reason**, considering the causes, a priori finds and determines what is **exact**; or, if Reason first finds what is exact, that finding must nevertheless be **received by sense and approved as concinnous** — lest either, with the **Pythagoreans**, we attribute too much to reason, or, with the **Aristoxenians**, too much to the senses. And that same opinion of his must again be inculcated, from bk. 1, ch. 2, where he says: *"It is the **harmonist's** aim everywhere to preserve the **rational positions of the Canon**, in no way repugnant to the senses (by most men's opinion); just as it is the **Astronomer's** aim to preserve the positions of the **celestial motions**, consonant with the observed revolutions — and these too taken, indeed, from the more evident and more universally apparent [phenomena]."* But if we should wish [to determine] the **intervals of the Planets** from the **rules** of harmony …

*[The catchword **"regu-"** (regu·**lis**, "rules") points to p. 531 (PDF 566), continuing the fifth reason, within Chapter VIII.]*

(printed p. 531 — Chapter VIII concludes: harmonic rules for the planetary intervals conflict with observation, so Scripture's and the Fathers' "Harmony of the heavens" must be taken metaphorically, not literally. Chapter IX asks whether the magnitude and density of the celestial bodies were determined harmonically, and Riccioli denies it, noting Kepler defined sizes geometrically and Rheita's proportions contradict observation. Chapter X then opens on whether the planets' motions were determined by harmonic proportions, beginning with Ptolemy's treatment in Harmonics 3.9-15.)

CHAPTER VIII

(conclusion)

…[But if we should wish to determine the intervals of the planets from the] **rules of harmony**, they will often be **repugnant to the observations** evidently made by sense; nor will the commensurations of the orbs, or the prosthaphaereses founded on them, **represent the places of the planets** such as are detected through Astronomical instruments and accurate observations; and they will so conflict as to be far from that nearness which this business would require, in order that **Reason might have a foundation**.

[We must say], rather, that **God willed to attain other ends through the celestial intervals and motions** — yet so that He also proposed **this** [end]: namely, the **beauty of the harmonic ratios**, to be wrought for Himself and to be contemplated by the intellectual creature. Since, therefore, observations **repeatedly cry out against and dissonate from the harmonic laws**, it must be said either that those ends could **not be acquired at once** through these means, or that **God truly did not will it**. Accordingly, the assertions of **Scripture, of the Fathers, and of the Wise** concerning the **Harmony and Concord of the heavens** are to be taken **not in a proper, or quasi-proper, sense** — as though nothing else were lacking to [the heaven] in respect of harmonic reckoning except a **sound sensible to us** — but in a **metaphorical sense**, and according to a certain **analogy and accommodation**. In the same way, too, that [text] of **Ecclesiasticus 32** must be understood, in political and economic governance: *"Have they made thee ruler? Be not lifted up,"* etc., *"and hinder not the music"* — that is, the **order and subordination of duties**, or the **peace and concord of citizens or of a household**.

CHAPTER IX

Whether the Magnitude and Density of the Celestial Bodies has been determined by Harmonic Proportions

(An Magnitudo et Densitas Corporum Caelestium ex Harmonicis Proportionibus Determinata fuerit)

[I.] This **we likewise deny**, for the reasons adduced in chapter 8 from number 8 [¶VIII], if what we said of the **intervals** be applied to the **Magnitudes and Densities** of the planets — **except**, however, [that here we lack] the argument *a posteriori*, taken from sensible experiment, concerning their **density**; for we have **no such experiment** by which we could convict of falsity those who might **feign the density and rarity of the planets from harmonic proportions**. Nor indeed did **Kepler** (*Epitome of Astronomy* bk. 4) define those [magnitudes and densities] from harmonic, but only from **Geometric**, reasons — as is clear from his own pages **484 to 489**: he only said that **Saturn is twice as high [far] as Jupiter, one-and-a-half times heavier, and one-and-a-half times rarer**, and so twice as high as it is heavier, and twice as ample as it is rarer, and proportionally of the rest; but that the **Sun is the densest of all**. (The proportions of density which he devises we have already reported, bk. 7, sect. 1, ch. 1, num. 7; and his various propositions bearing on it we reviewed, bk. 7, sect. 6, ch. 8, num. 16 — to which we refer the Reader, but so that he also consult Scholium 2 of the same chapter.) The same Kepler, moreover (*Harmonics* bk. 5, ch. 4), **expressly denies** that the proportions of the bodies of the planets are harmonic.

But as for what **Fr. Anton Maria de Rheita** asserts (in his *Oculus Enoch et Eliae* bk. 4, ch. 2, member 4) — that the **Earth is to the Sun as a square root to its square**, or as **10 to 100**, so that the **Sun is a hundredfold greater than the Earth**, and the like (of which we treat, bk. 7, sect. 6, ch. 8, schol. 3) — whence **Kircher** (*Musurgia* 10, p. 379) says that, **if they were true, certain harmonic proportions could be gathered** [from them]: this, I say, **by no means agrees** with the more exact observations of the diameters and distances held by us and by others (as is clear from what was said, bk. 7, sect. 6, from ch. 10). And Kircher himself (same page, and p. 381) holds the **observations of Rheita suspect**, on which he built those proportions, and says: *"But I greatly doubt that Rheita, for the apparent quantity of the said diameters*

assumed at his pleasure, ***chose so beautiful and specious a proportion rather than [actually] observed it****."*

CHAPTER X

Whether and by what reckoning the Motions of the Planets have been determined by God from Harmonic Proportions

(An et Qua Ratione Motus Planetarum ex Proportionibus Harmonicis Determinati fuerint a Deo)

[I.] Here, indeed, **Johannes Kepler triumphs** (*Epitome of Copernican Astronomy* bk. 6, pp. 477, 501, and 900; *Mysterium Cosmographicum* ch. 14, 20, 21; but **chiefly** in his book of *Harmonics*) — in whose preface he relates that he **began to speculate on this matter 22 years before**, and spent the best part of his life in Astronomy to this end: namely, **to show that the whole nature of Harmony, however great it is — with all its parts expounded in book 3 — is to be found among the celestial motions**. From Kepler a few things were selected by **Pierre Hérigone** (*Cursus Mathematicus* vol. 5, from p. 573) and by **Mersenne** (on Genesis 4, p. 1558, etc.); but more fully by **Athanasius Kircher** (*Musurgia* bk. 10, p. 376).

[Margin: Ptolemy's doctrine on the harmony of the celestial motions.]

But **before Kepler, Ptolemy** (*Harmonics* bk. 3, from ch. 9) took up this very argument for himself to treat — whom, therefore, it is fitting to hear first.

[II.] Ptolemy, then (*Harmonics* bk. 3), [teaches as follows]:

- **(ch. 9 — the motion in longitude** [the diurnal motion]): he **likens the Rising and Setting of the stars to the gravest voices, and the Meridian ascents to the acutest** — both because, just as those who train the voice begin from the gravest, and when they have ascended to the acutest, descend until they end in the gravest (and so are moved **from silence, through the acutest voice, to silence**), so the stars pass from rising, through the meridian, to setting, from one privation of aspect to another; and because the **lower places** (to which the points of rising and setting are likened) render the **gravest** sounds, but the **higher**, like the meridian, the **acutest**.
- **(ch. 10 — the motion in altitude** [distance]): in the motion at the **maximum distance** from the earth the **Diatonic** genus reigns (because in it two intervals are not less than the rest); in the **minimum distance**, the **Enharmonic** (because in it both [outer] intervals are less than the rest); and in the **mean distance**, the **Chromatic** (because in it the Lichanos, the index-string, holds the middle of the tetrachord, standing midway between the Diatonic and the Enharmonic).
- **(ch. 11 — the motion in latitude**): he compares it to the changes that occur among the modes (the transgression of the genera): the **Mixolydian and Hypodorian** (the extreme modes) answer to the **maximum latitude** toward North and South; the [Dorian], the middle, to when the planets **lack latitude** and are on the Ecliptic (or in declination, or on the Equator).
- **(ch. 12 — the aspects to the Sun**): the remaining order of the tetrachords he likens to the remaining aspects — the tetrachords of the **disjunct** strings (*diezeugmenon*) to the intervals from **occultation to the appearances** of the stars and to the **oppositions with the Sun** (acronychal risings and full moons); the **conjunct** tetrachords (*synemmenon*) to the **quadratures**; and so of the rest.
- **(ch. 13 — the 360-degree circle**): the whole circle divided into 360 parts, he assigns to the **Trine** aspect 120 parts, to the **Quadrate** 90, to the **Sextile** 60, and to the **Opposition** 180; the Proslambanomenos being taken at 180 parts… [he gives, of the middle strings, 20; to the last *diezeugmenon* 90; to the last *hyperbolaeon* 60].

• **(ch. 14)**: he inquires into the consonances **Diapente, Diatessaron**, and some others, through the division of the circle into 360 parts — as certain aspects are of the whole circle, or of a portion of it due to another aspect — yielding a sesquitertian, sesquialteran proportion, and the like.

• **(ch. 15)**: finally he says that the **sound of Saturn and Mars** (the **malefics**) makes a **Diatessaron** consonance with the sound of the **benefics** — namely, that the last [string] of the *hyperbolaeon* (the "excellent") of **Saturn** consonates with the last of the *diezeugmenon* (the "disjunct") of **Jupiter**; the last of the *synemmenon* (the "conjunct") of **Mars** with the **Mese**, or middle [string], of **Venus**; and that the sound of **Saturn** belongs rather to the **Solar sect**, [and that of Mars …]

*[The catchword **"tis"** (Mar·**tis**) points to p. 532 (PDF 567), continuing Ptolemy's doctrine and then Kepler's, within Chapter X.]*

(printed p. 532 — Chapter X continued, the heart of Kepler's celestial harmony. Ptolemy's doctrine ends with Riccioli's verdict that he partly plays the poet; then Kepler's own doctrine follows: the Sun as "Choragus" dividing the circle into 720 harmonic parts, and the Third (Harmonic) Law — periodic times as the 3/2-power of mean distances, confirmed by Wendelinus for Jupiter's satellites. Kepler's tables of apparent diurnal motions then show that the convergent motions of adjacent planets form near-perfect consonances.)

CHAPTER X

(continued — Ptolemy concluded, then Kepler's harmony of the celestial motions)

…[and that the sound of Saturn belongs rather to the Solar sect, but that] of **Mars to the Lunar sect**; wherefore all the **configurations of Saturn to Jupiter are beneficent**, but of the aspects of **Saturn to the Sun only the Trines** are beneficent (as more consonant than the rest); so too only the **trine configurations of Mars to Venus and the Moon** are beneficent; further, the configurations of **Saturn to the Moon and Venus are evil**, and those of **Mars to the Sun all dangerous**. Let it suffice us to have indicated these, so that it may appear that **Ptolemy** in these matters **partly rather plays the Poet** (as seemed also to Kepler, in the appendix to his *Harmonics*), and partly, out of the **Astrological faculty or vanity, hunts for a harmony in the heaven**.

[Margin: The proportion between the Periods and the Distances of the Planets.]

[III.] But **Kepler** (*Epitome of Astronomy* bk. 4, p. 477; and *Harmonics* bk. 3, ch. 6) teaches that the **least number suitable for determining all the parts of the Monochord** — for constituting the system of the **double Diapason**, that is, of the soft and hard song [minor and major] — is **720**; and since, from the ancient observations of Aristarchus and from more recent ones, the **apparent diameter of the Sun at apogee subtends 30 minutes** (he says), which is **1/720 of the whole circle**, this diameter was determined of such a quantity that the first body — that is, the **Sun, the Choragus [choirmaster] of the celestial music** — should divide the circle, for the earth-dwellers and the contemplating creature, **according to the Harmonic laws: that is, into 720 parts**. Which number can be divided into very many aliquot parts — by 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, and 48 — that is, in order, into the parts **360, 240, 180, 144, 90, 80, 72, 60, 48, 45, 40, 36, 30, 15**. (The consonances he determines from the sides of the figures inscribable and demonstrable in the circle we have already taught — ch. 4, Scholia 1 and 2; and *Harmonics* bk. 4, ch. 5 and 6; and *Epitome* bk. 6, p. 901.) He teaches that **sublunary nature is so often sensibly stirred and stimulated to act**, as often as the **planets are configured harmonically among themselves** — that is, when their aspects and radiations occur at such a distance from one another under the Zodiac as the harmonic proportions require (of which Aspects we shall speak in the following chapter; for now we treat only of the **motions** themselves).

[Margin: The proportion between the periods and the distances of the planets (Kepler's Third Law).]

Of these, Kepler treats (*Epitome* bk. 6, from p. 90, and in the whole of *Harmonics* bk. 5, especially ch. 3), where he affirms, **as a most certain thing, that the proportion between the periodic times of any two Planets is precisely the sesquialteran [3/2-power] proportion of their mean distances from the Sun** — provided the mean be the arithmetic mean between the two diameters of the elliptic orbit, which is a little less than the longer diameter. So that, for example, if from the **period of the Earth** (which is **1 year**) and from the **period of Saturn** (which is **30 years**) you take the **third part of the proportion** (that is, the **cube roots**), and then make the **double of this proportion** (by squaring the roots), there will come forth the most exact proportion of the distances of Saturn and the Earth from the Sun. For the cube root of 1 year is 1, and its square 1; but the cube root of 30 years is a little more than 3, and its square a little more than 9: therefore **Saturn's mean distance from the Sun is a little more than ninefold the mean distance of the Earth**. This Keplerian proposition **Hérigone** also accepts (*Cursus Mathematicus* vol. 5, p. 573); and the same proportion between the **periods and the distances of the Satellites of Jupiter** was noted by **Wendelinus** [Govaert Wendelen] in a most learned **letter written to me by himself**. Kepler proceeds, and (ch. 4) confesses that, in the **periodic times of the planets compared among themselves, there are no harmonic proportions** — to which **Mersenne** (on Genesis 4, p. 1558) and **Kircher** (*Musurgia* 10, p. 377) readily subscribe. Now the **periodic motions of the planets around the Sun** are gathered, from all the motions through all the degrees of the whole circuit (long, mean, and small), by Kepler, as in the following table.

The apparent diurnal motions of the planets, and the "song" of each

(Motus Apparentes Diurni — each planet's daily angular motion at aphelion and at perihelion, in arc-minutes ′ and seconds ″; and the consonance traced by its own motion-range)

Planet	Aphelion motion	Perihelion motion	Index-letters	Own proportion	Consonance
Saturn	1′46″	2′15″	a, b	4 : 5	major Third
Jupiter	4′30″	5′30″	c, d	5 : 6	minor Third
Mars	26′14″	38′1″	e, f	2 : 3	Diapente (fifth)
Earth (*Tellus*)	57′3″	61′18″	g, h	15 : 16	Semitone
Venus	94′50″	97′47″	i (n), k	24 : 25	Diesis
Mercury	164′0″	384′0″	l, m	5 : 12	Diapason with minor Third (octave + minor third)

*(This is the famous result of Kepler's Harmonices Mundi: each planet's own range of angular speed, from slowest at aphelion to fastest at perihelion, spans a musical interval — Saturn a major third, Jupiter a minor third, Mars a fifth, the Earth a semitone, Venus a barely-perceptible diesis, Mercury an octave-and-a-third. Riccioli distinguishes these **apparent** diurnal motions from the **mean** diurnal motions, which differ by a few seconds and from which the consonances are exactly computed: e.g. Saturn 1'48":2'15" = 108:135 = 4:5; Earth 57'28":61'18" = 15:16; Mercury 164':394' = 5:12. All six check.)*

Harmony of Pairs

*(Harmonia Binorum — the harmonic ratio between the extreme diurnal motions of each adjacent pair of planets: **Divergent** = aphelion-motion of the upper to perihelion-motion of the lower; **Convergent** = perihelion of the upper to aphelion of the lower)*

Adjacent pair	Divergent	Convergent
Saturn–Jupiter	a : d = 1 : 3	b : c = 1 : 2 (Diapason / octave)
Jupiter–Mars	c : f = 1 : 8	d : e = 1 : 5
Mars–Earth	e : h = 5 : 12	f : g = 2 : 3
Earth–Venus	g : k = 3 : 5	h : i = 5 : 8
Venus–Mercury	i : m = 1 : 4	k : l = 3 : 5

[Margin: Explanation of the preceding table.]

In the preceding table, then, the **first column** ["Harmony of Pairs"] indicates the harmonic proportions between the diurnal motions of two planets around the Sun — whether **divergent** (comparing the **aphelion of the upper with the perihelion of the lower**) or **convergent** (the **perihelion of the upper with the aphelion of the lower**); to mark which, the **alphabetic letters** are added, which, sought in the **second column** [the motion-table], signify the aphelion or perihelion motion of the planet. For example, in the first column under the title "divergent" you see **a and d**, and opposite to them this fraction **1/3**; now **a**, in the second column, signifies Saturn's **aphelion** motion, and **d**, Jupiter's **perihelion**; therefore between the diurnal motion of Saturn's aphelion (which is **1′46″**) and Jupiter's perihelion (which is **5′30″**) there is a proportion as of **1 to 3** — for, the motions being resolved into seconds, they are, in diurnal motions, **Saturn's 106″, Jupiter's 330″**, between which is the proportion as 1 to 3. But between **b and c** — that is, between the **convergent** diurnal motions of Saturn's perihelion and Jupiter's aphelion — the proportion indicated by the fraction **1/2** (namely double, or as 2 to 1); for Saturn's perihelion motion is **135″** and Jupiter's aphelion **270″**, between which is a most perfect **Diapason** [octave].

But in the others — **except for Jupiter with Mars** — the proportions of the motions are **so near to the harmonic** that, if strings were so tuned, the ears could not easily discern the imperfection of the consonance. So Kepler concludes that there are **perfect harmonies**: between **Saturn's perihelion and Jupiter's aphelion**, a **Diapason** [octave]; between **Jupiter's perihelion and Mars's aphelion**, nearly a **Diapason-with-soft-third** [octave + minor third]; between **Mars's perihelion and the Earth's aphelion**, a **Diapente** [fifth]; between the **perihelia of the same** [Mars and Earth], a **soft [minor] Sixth**; between the **aphelia of the Earth and Venus**, a **hard [major] Sixth**; between the **perihelia of the same** [Earth and Venus] …

*[The catchword **"Peri"** (Peri·**helios**) points to p. 533 (PDF 568), continuing Kepler's pairwise harmonies, within Chapter X.]*

(printed p. 533 — Chapter X concludes and Chapter XI opens. Kepler's harmony finishes: the motion-proportions yield musical modes and four voice-parts (Bass to Saturn and Jupiter, Soprano to Mercury), with the Sun as the "Regia" of nature perceiving these harmonies — whence Kepler confirms heliocentrism. Riccioli's verdict: more ingenuity than solid doctrine, resting on rejected heliocentrism; the celestial harmony of Scripture is only analogical and metaphorical. Chapter XI then opens on the force of

astrological Aspects, with Kepler deriving 13 efficacious aspects from inscribed polygons.)

CHAPTER X

(continued — Kepler's harmony concluded, and the Author's verdict)

…[between the] **perihelia of the same** [Earth and Venus], a **soft [minor] Sixth**; and between **Venus's aphelion and Mercury's perihelion**, or even between their **perihelia**, a **Disdiapason** [double octave]. From these and other considerations — but not without many cautions — (ch. 5) he tries to **drag the proportions of the planetary motions to the places of the System, or to the keys of the Musical Scale**, in the genus of **hard and soft song** [major and minor]; and (ch. 6) that, in the extremes of those same motions, there are **expressed by God, in some way, Musical Tones or Modes**; and (ch. 7) that there are given **universal Harmonies of all six planets, as it were common Counterpoints, in four forms**; and (ch. 8) — although he confesses that **in the heaven there are neither sounds, nor any motions (in which he considers Harmonies) that are real, but only apparent from the Sun**, and that there is **no solid natural cause for comparing the apparent motions of the planets with human voices** — yet, by I-know-not-what enticement of congruence and analogy, he assigns the **Bass to Saturn and Jupiter, the Tenor to Mars, the Alto to the Earth and Venus, the Descant [Soprano] to Mercury**.

Hence, having made a step to the **Eccentricities** (in the very prolix ch. 9, with many axioms — for the most part feigned — ingeniously coordinated to his purpose), he tries to show that the **eccentricities of the orbs had to be determined from Harmonic reasons**, so that the extremes of the aphelion and perihelion motions might represent the harmonic proportions [given] above; and that therefore the **inscriptions and circumscriptions of the orbs in (or about) the five Regular bodies had to yield to these harmonic reasons**, and that the intervals of the planets could not be so exactly constructed from the Regular bodies [alone], **lest the Harmonies of the extreme motions about the Sun should perish**. Finally (ch. 10), since he seems to himself to have grasped a **Harmony among the extreme motions of the planets — not real, but apparent to the Sun, or seen from the Sun** — he concludes that the **Sun is the Royal seat [Regia] of all nature**, and that there is in it **some Mind, hidden from us, which can perceive those harmonies** (since they arise only from the motions subtending angles at the Sun). Hence he tries to confirm the **rest of the Sun and the motion of the Earth** — without which a great part of those harmonies perishes. Yet the same [Kepler] taught how to investigate the harmonic proportions in the motions seen **both from the Sun and from the Earth** (*Epitome* bk. 6, p. 901) — which doctrine we set forth from him on another occasion (bk. 7, sect. 5, ch. 8, num. 7), so that we need not repeat it here.

[Margin: Our Opinion.]

[IV.] Our Opinion, however — from which **Mersenne** (on Genesis 4, from p. 1558, and p. 1704) and **Kircher** (*Musurgia* 10) are not far — is that the aforesaid endeavors of Kepler contain **more ingenuity than solid erudition or true doctrine**. For, **first**, a great part of them rests on the **immobility of the Sun at the center of the World**, on a certain **mental force of the Sun apprehending the harmonies**, and on the **annual motion of the Earth about the Sun** — which hypothesis we have already rejected in the last chapters of the preceding Section. **Secondly**, the three former reasons (adduced ch. 8, num. 8) militate here. **Thirdly**, since Harmony properly so called (harmonic proportions) is found **neither in the periods of the planets compared among themselves, nor in most of the diurnal motions seen from the Earth** (where the contemplating creature of the divine works is, accustomed to sensible harmonies), **nor in the motions as to latitude, nor among the extreme motions themselves apparent from the Sun so exactly** as was fitting and possible for God (if, in determining the motions, He had had the harmonic reasons set before Him in the **Archetype**); and finally, since those extreme motions of two planets (e.g. the perihelion of Saturn and the

aphelion of Jupiter) **very rarely concur**, but for the most part the motions proceed **without these harmonies** — it seems rather to be asserted that **the Harmony which Scripture and the Fathers and very many wise men recognize in the heaven is to be understood only analogically and metaphorically**, by a certain accommodation and likeness: namely, that just as in **sensible Harmony**, out of unequal and diverse sounds and voices, there arises a concord pleasant to the ears, so out of the motions of the celestial bodies — though diverse and unequal — there follows nevertheless an **admirable order toward the end sought by God**, and a **conspiration of the means toward the same**, most delightful to the Angelic and Human intellect contemplating these more deeply. But the other things, sought out with such ingenuity by Kepler and others to establish some such Harmony in the heavens (so that nothing should be lacking to it but a sensible sound), seem **mere symbolisms, poetic or rhetorical rather than philosophical** — as **Kepler** himself, in the appendix to his *Harmonics* (p. 253), speaks of the symbols of **Ptolemy** and of **Robert Fludd** ("of the Tides"); and **Mersenne** (on Genesis 4, p. 1558) judged the same of **Kepler's** analogies, calling them symbolisms, **at most Oratorical** — such as Orators could use to amplify, by such metaphors, the **Divine Providence in celestial things**. And [Kepler] adds that the **eternal Geometry of God is concerned with harmonic proportions** rather than with the temperings of figures, colors, tastes, and odors; and that, just as we cannot — by the image of the divine Geometry impressed on our mind — give a reason **why this figure, or this mixture of colors, should rather delight the eyes**, or why these tastes or odors should please the palate or nostrils more than others (nor in these must one recur to the image of Geometry), so neither, in giving a reason **why these sounds rather delight the hearing**; and much less, from what is pleasant to the hearing alone, is a reason to be drawn of the celestial motions and of the **intelligible beauties**.

CHAPTER XI

Whether, and what, force and determination the Aspects of the Stars have from Harmonic Configurations

(An et Quaenam Siderum Aspectus vim habeant ac determinationem ex Configurationibus Harmonicis)

[I.] Much about this argument I taught (bk. 7, sect. 5, ch. 8), since the **aspects are affections of the planets in longitude**, of which that section treated; and I seemed there to report the doctrine on this matter in such a way as **not to disapprove it**, because the **divisions of the circle by aspects seem to have a greater kinship with the harmonic consonances** than the motions of the planets do. Moreover **Kepler** taught (*Harmonics* bk. 4, ch. 5, props. 1, 2, 3) that the kinship of the **Radiations (or Aspects)** with the circle, its arcs, and the figures inscribable in it is **greater** than that of the Consonances; that the **congruence** of [the figures] inscribable in the circle avails more toward constituting **efficacious configurations** than toward consonances; and that congruence avails more toward the same than does **knowability** [scibilitas]. (What congruence is, and what the knowability of figures is, is indicated here in the Scholia of ch. 4.)

[Margin: The efficacious configuration.]

Further (ch. 5), by Kepler an **efficacious configuration** is defined: [when] the **radii of two planets make such an angle** as is apt to **stimulate sublunary nature and the lower faculties of living things**, so that each is the more **excited about its own work at the time of the configuration**. Then he assumes an **axiom**: that the **arc of the Zodiac-circle which the side of a congruent and knowable figure (or star-figure) measures out is the module of an efficacious configuration**, and that the **angle of a knowable and congruent figure is the module or measure** of an efficacious configuration. These being posited — since he had demonstrated (bk. 1) that the **Diameter of the circle, the Tetragon [square], the Trigon [triangle], the Hexagon and Octagon and octagonal star, the Dodecagon and dodecagonal star, the Pentagon, the**

Decagon, and the pentagonal and decagonal stars are **knowable**, and (bk. 2) **congruent** — he therefore constituted **13 efficacious configurations**, of which:

• the most efficacious is the **Conjunction**, to which corresponds the whole circuit of the circle, **360 degrees**;

• next, the **Opposition**, because [its lines] meet in one same line (which is the most perfect congruence), to which corresponds the **semicircle, 180 degrees**;

• next, the **Quadrate** [Square] aspect, to which corresponds the **quadrant, 90 degrees**;

• after it, the **Trine** (or Trigon), to which corresponds the **third of the circle, 120 degrees**;

• then the **Sextile** (or Hexagon), to which corresponds the **sixth of the circle, 60 degrees**;

• then the remaining Keplerian aspects, in that order of efficacy which we present at once in a table.

[The first paragraph ends introducing the table of Kepler's 13 efficacious aspects, which follows on p. 534 (PDF 569), within Chapter XI.]

(printed p. 534 — Chapter XI continued, Kepler's harmonic theory of the astrological Aspects. The table of 13 efficacious configurations is given; then Kepler's earlier axiom that the aspects answer to the consonances below the octave, which he himself refutes from meteorological experience and corrects: the aspects answer to the consonance of the aspect-arc with the whole circle. Riccioli judges the old axiom not merely insufficient but false, the comparison of consonances with aspects being geometrically unsound.)

CHAPTER XI

(continued — Kepler's harmonic theory of the Aspects)

Efficacious Configurations

(Configurationes Efficaces, from Kepler's Harmonics bk. 4, ch. 5) — the 13 aspects, the inscribed "knowable and congruent" figure whose side generates each, and the consonance Kepler once attached to it

Aspect	Zodiac arc (°)	Symbol	Generating figure	Harmony once believed by Kepler
Conjunction	0 / 360	conjunction	(the whole circle)	—
Opposition	180	opposition	Diameter of the circle	Diapason
Quadrate (Square)	90	square	Tetragon (square)	Diatessaron
Trine	120	triangle	Trigon (triangle)	Diapente
Sextile	60	sextile	Hexagon	Semiditonus (minor third)
Octile (Semiquadrate)	45	—	Octagon	Hexachordum minus (minor sixth)
Trioctile (Sesquiquadrate)	135	—	Octagonal star	—
Semisextile	30	—	Dodecagon	—
Quincunx	150	—	Dodecagonal star	—
Quintile	72	—	Pentagon	Ditonus (major third)
Tridecile (Sesquiquintile)	108	—	Decagonal star	—

Aspect	Zodiac arc (°)	Symbol	Generating figure	Harmony once believed by Kepler
Biquintile	144	—	Pentagonal star	Hexachordum maius (major sixth)
Decile (Semiquintile)	36	—	Decagon	—

(Each aspect's arc is what the side of the named regular polygon (or star-polygon) subtends in the circle: the diameter [2-gon] gives 180°, the square 90°, the triangle 120°, the hexagon 60°, and so on; the star-polygons give the "skip" arcs (the octagonal star 135°, the pentagonal star 144°, etc.). The last column shows the consonance Kepler had earlier matched to seven of these figures.)

[Margin: An old axiom of Kepler, condemned by Kepler himself.]

[II.] The same **Kepler** (*Harmonics* bk. 4, ch. 6) treats of **Astrology** — of which he had written in his book *On the New Star* (ch. 8, 9, 10), and in his responses to the physicians **Helisaeus Roslin** and **Philip Feselius**, who attacked these new aspects; and finally [what] he had said of them in his *Ephemerides* (pp. 33–36). For in the year **1606** he had assumed as an **axiom** that **God the Creator drew the law of ordering the Aspects from the Harmonies of song below the octave**, or that He **attuned the ears of men to the celestial Aspects, [making them] judges of those concordances** — an axiom coined by no one but himself (as appears from p. 34 of the *Ephemerides*), which he **now refutes**: because [if it were true] there would have to be **as many aspects as there are simple consonances up to the Diapason**. For [then] the **Quadrate** aspect should answer to the **Diatessaron**; the **Trine** to the **Diapente**; the **Opposition** to the **Diapason**; the **Quintile** to the **major Third (Ditone)**; the **Sextile** to the **minor Third (Semiditone)**; the **Biquintile** to the **major Sixth (greater Hexachord)**; the **Sesquiquadrate** to the **minor Sixth (lesser Hexachord)**. For if you take from the whole string as great a portion as each aspect takes from the circle, the **residue of the string makes, with the whole string, the consonance assigned to that aspect**.

But, says Kepler, in his **Meteorological observations** it was found that **sublunary nature is stirred even by the Semisextile** aspect (which intercepts a twelfth of the circle), although, a twelfth of the string being removed, the residue of eleven parts does **not** consonate with the whole; and, on the contrary, that nature is **not** sensibly stirred by the **Sesquiquadrate** (which intercepts three-eighths, or 135°), although, three-eighths of the string being removed, the remaining five parts **do** consonate with the whole. Hence, stirred [by this], he **corrects his axiom**, and teaches that the proportion of the aspects answers not to the major concordances [by the residue] but **to the consonance of the aspect-arc with the whole circle**. For example, the **Trine** does not answer to the Diapente, but to the **Diapason-Epidiapente** [octave + fifth = a twelfth, 1:3]; for the Trine is between planets distant a **third** of the Zodiac (120°), not the residue (240°), and **between 3 and 1** is the consonance Diapason-Epidiapente. The **Quadrate** answers not to the Diatessaron, but to the **Disdiapason** [double octave, 1:4], being between [planets] distant **90°** (a fourth), and between 4 and 1 is the Disdiapason. The **Quintile** is between [planets] distant **72°** (a fifth), and between 5 and 1 is the **Diapason-with-Ditone** [octave + major third, 1:5], which not all admit among the true consonances. So too the **Sextile** answers not to the minor Third but to the **Disdiapason-Epidiapente** [1:6]; the **Biquintile** not to the major Sixth (as we supposed above) but to a [consonance] **composed of the major Third and the Diapason**; the **Sesquiquadrate** not to the minor Sixth but to one **composed of the Diatessaron and the Diapason** — as is clear from the proportion of the part to the whole (the Sextile distant a sixth of the Zodiac; the Biquintile two-fifths = 144°; the Sesquiquadrate three-eighths = 135°): for between **6 and 1** is the Disdiapason-with-Diapente; between **5 and 2**, the Diapason-with-Ditone; between **8 and 3**, the Diapason-with-Diatessaron. Let the **preceding table be emended**, then, as in the following, to which **we have added three consonances omitted by Kepler**.

The Aspects and their consonance with the whole circle

(the corrected table — Aspect · intercepted Zodiac degrees · consonance of the arc with the whole circle of 360°)

Aspect	Zodiac arc (°)	Consonance with the whole circle (arc : 360°)
Opposition	180	Diapason (1 : 2)
Quadrate	90	Disdiapason (1 : 4)
Trine	120	Diapason-Epidiapente (1 : 3)
Sextile	60	Disdiapason-Epidiapente (1 : 6)
Octile	45	— (a cross; no received consonance)
Trioctile	135	Diapason-Diatessaron (3 : 8)
Semisextile	30	— (a cross)
Quincunx	150	Diapason-with-Semiditone (5 : 12)
Quintile	72	Disdiapason-with-Ditone (1 : 5)
Tridecile (Sesquiquintile)	108	Diapason-with-major-Hexachord (3 : 10)
Biquintile	144	Diapason-with-Ditone (2 : 5)
Decile (Semiquintile)	36	— (a cross)

(The consonance is the ratio of the aspect's arc to the whole 360°, all verifiable — 180:360 = 1:2 (octave); 90:360 = 1:4 (double octave); 120:360 = 1:3 (twelfth); 60:360 = 1:6; 135:360 = 3:8 (eleventh); 150:360 = 5:12; 72:360 = 1:5; 108:360 = 3:10; 144:360 = 2:5. The three crosses (Octile 1:8, Semisextile 1:12, Decile 1:10) mark arcs giving no received consonance. The Conjunction (360°, the whole circle) heads the list without a proper ratio.)

[Margin: The falsity of the aforesaid axiom.]

[III.] These and other things — **having become more cautious than himself** — Kepler teaches in that ch. 6; among which the notable ones are these, which it pleases [me] to state in his words, with our little explanation added… [namely, that] a finite **straight line**, truncated or prolonged, **remains a straight line**; but a **circle truncated does not remain a circle**: whence it follows that a **proportional section of two straight lines** is possible, but **not of two arcs of one circle**. … That a true mathematical and causal comparison of the **Concordances with the Aspects** may stand, [the old axiom] must plainly **be overthrown** — since it is not only **insufficient**, but [also false] …

*[The catchword **"Con"** points to p. 535 (PDF 570), continuing Riccioli's discussion, within Chapter XI.]*

(printed p. 535 — the last page of Book IX. Chapter XI concludes: Kepler distinguishes consonances from aspects as two nations sprung from the same fatherland, Geometry — his notions of a world-soul Riccioli flags as inadmissible without correction — and his final three-order ranking of the aspects is tabulated. Riccioli's closing judgment: since several aspects answer to no consonance, this harmony was not God's rule; celestial "music" is metaphor and analogy only. The book ends with FINIS.)

CHAPTER XI

(conclusion, and the end of Book IX)

…[Con]sult now, in our chapter 4, the **first Table of Consonances**, and you will see that there are **not a few [consonances] to which no aspect is here attributed**, and in turn that there are some **aspects here to which the crosses appended indicate that no Consonance answers**.

[Margin: Kepler distinguishes the Aspects from the Consonances.]

[IV.] These and the like **compelled Kepler** to confess (in the same bk. 4, ch. 6) that the **harmonic consonances and the Aspects** have indeed something in common as to **origin** — from the **divisions of the circle** — but that **Music and Meteorology [astrology] are born thence in a different manner**; for **diverse causes concur to constitute the Aspects**, and **Nature has a choice of those [aspects] that are furnished with more prerogatives**. But let his words be noted, I pray: *"What, then, is that which sets a limit to the number of the aspects? And why is no **Semiquadrate, or Octile**, no **Decile or Tridecile**, introduced except only **after the principal ones**? Why is the **Sesquiquadrate**, ennobled by musical kinship, either omitted or held ignoble — while the **Semisextile**, a stranger in Music, is not only inserted, but even displayed among the first?"* He at once answers himself: ***"Because it is not Music that forms the Aspects, but Geometry [that forms] both kinds** — yet the one by some laws, the other by others. For whatever is both **Harmonic in Music and Efficacious in Meteors** comes from a **noble figure** that has some singular privileges in Geometry. But **Meteorology and Music are diverse, like two nations sprung from the same fatherland, Geometry."***

And he adds that the **harmonic proportions, arisen from the circle, nevertheless led out colonies that departed from the circle**; but that the **Aspects remained within the fatherland of the circle**, and use no other laws than the circle's — taken from the plane figures inscribed in the circle, **Regular and Congruent**. The other things which (ch. 7) [he treats] — concerning a **faculty of sublunary nature**, and a certain **Soul of the whole universe**, and a **Soul of the Earth** that would perceive the force of the aspects and be stirred to act — are **neither of this place, nor to be admitted by Catholics without correction**. Yet here we must select, from bk. 6 of the *Epitome of Copernican Astronomy*, the aspects which Kepler finally accepted, and in that degree of dignity which he afterward recognized in them by the very observation of meteorological effects.

[Margin: the falsity of the aforesaid axiom.]

[V.] Therefore (*Epitome* bk. 6, p. 843) he teaches that in the **first degree** of aspects are placed: the **Conjunction**, as the principle of all; the **Opposition**, as occurring in all three divisions of the circle; the **Quadrate**, as occurring in two (the area of its figure being **effable** [expressible]); the **Sextile**, because its side is effable; the **Semisextile**, because its side is among the ineffable of a more perfect order, and because, twelve times repeated, it encompasses a stable plane; and the **Trine**, since its side is effable in power — for which, consult what was said in Scholium 1 of ch. 4. In the **second order** he says are the **Quintile and Biquintile** (because, though their sides are ineffable of a worse order, they share among themselves a **divine proportion**, and their figures excel in congruence into solid figures), and the **Quincunx** (because its figure is fruitful in the congruence of planes). But the **Decile and Tridecile** already fall short of congruence; and the **most ignoble are the Octile and Sesquiquadrate**, because they are formed from sides neither effable nor of divine proportion — and though these degrees are not altogether indubitable, let this stand as the final table

of the Aspects, according to Kepler's mind:

The Order of the Aspects in Force and Dignity

(Ordo Aspectuum in Vi et Dignitate — per Kepler; aspect · intercepted Zodiac degrees · consonance of the arc with the whole circle)

Order	Aspect	Zodiac arc (°)	Consonance with the whole circle
First	Conjunction	0 / 360	Unison
	Opposition	180	Diapason
	Quadrate	90	Disdiapason
	Sextile	60	Disdiapason-Epidiapente
	Semisextile	30	none
	Trine	120	Diapason-Epidiapente
Second	Quintile	72	Disdiapason-with-Ditone
	Biquintile	144	Diapason-with-Ditone
	Quincunx	150	Diapason-with-Semiditone
[Lesser]	Decile	36	none
	Tridecile	108	Diapason-with-major-Hexachord
Most ignoble	Octile	45	none
	Sesquiquadrate	135	Diapason-Diatessaron

It deserves consideration, indeed, that the **other consonances** to which some aspects answer are either a **principle or a species of the Diapason** — which Kepler did not notice; and that, among the consonances pertaining to the Diapason, enumerated in the table of ch. 4, there is **not one which has its own Aspect in the Zodiac**, except the one composed of the **Diapason and the minor Hexachord** (as is between 16 and 5) — and that only approximately, since 360 is not divided into 16 whole parts (a 16th of it is 22½, and five such make 112½, a number near the 108 of the Tridecile, which pertains to the Diapason-with-major-Hexachord); no wonder, then, if their efficacies cannot be told apart.

Nor, however, do I think that this [agreement] with the Diapason was **proposed to God as the mode of determining those aspects** — since, even without it, and **without any consonance whatever, three aspects are given**: namely the **Semisextile, the Decile, and the Octile**. But just as, in the mixture of colors, tastes, odors, and temperaments — in innumerable plants and animals, in the **climacterics, in the rhythms of the pulse, in the kinds of fevers**, and in very many other things — there are **certain degrees expressible by number** (were their nature perfectly known); and yet, because those degrees **can** bear the proportions due to harmonic numbers, we are **not therefore bound to be anxious about hunting out a Music in them beyond the looser bounds of Metaphor and Analogy** — so neither in the **Periods of the planets, nor in their Motions, nor in their Bulks, nor in their Intervals**: as though God had wisely made nothing except what He had subjected to the laws of Harmony.

FINIS — The End of Book IX

(LIBRI NONI FINIS)

(So ends Liber IX of the Almagestum Novum — Riccioli's "De Mundi Systemate." This Section V, "On the Harmonic System of the World," closes the book. The catchword "LIBER" at the foot of the page points to Liber X (Book X), which begins a new book beyond this Section.)

[Translator's note on the final sentence: the Latin reads "Quasi nihil à Deo factum sit sapienter, nisi quod Harmoniae legibus non subiecerit" — literally awkward (with a "non"); the sense, fitting the whole argument, is the ironic reductio: it is absurd to suppose that whatever God did NOT make conform to musical harmony was therefore made unwisely. God's wisdom shines in the world's order and proportion — but that order need not be a literal, audible, or strictly musical harmony.]